High Energy Astrophysics Dr. Adam Ingram

## Schedule

- Lecture 1: Shocks - Blast waves from huge explosions, strong shock conditions at the front of the blast wave.
- Lecture 2: Shock Acceleration - How electrons are accelerated to ultra-relativistic energies at strong shock fronts.
- Lecture 3: Synchrotron Radiation - The emission mechanism.
- Lecture 4: Synchrotron Radiation - Synchrotron self-absorption and spectral ageing.
- Lecture 5: Black Holes - GR crash course, ISCO, Eddington luminosity
- Lecture 6: Accretion discs - Structure, luminosity and spectrum.
- Lecture 7: The X-ray Corona - Thermal Compton up-scattering, Xray reflection.
- Lecture 8: Black Hole Mass and Jets - The AGN zoo, evidence for black holes, jet mechanisms, super-luminal jet motion.
- Lecture 9: Gravitational Waves - Derivation, measurements, implications.
- Lecture 10: Galaxy Clusters - Thermal bremsstrahlung radiation, Sunyaev-Zeldovich effect.


## Reading Material

- "High Energy Astrophysics" by Malcolm Longair
- "Accretion Power in Astrophysics" by Frank, King \& Raine
- "Radiative Processes in Astrophysics" by Rybicki \& Lightman
- "Gravitational Wave Radiation by Binary Black Holes" by Ryan Rubenzahl (https://rrubenza.github.io/project/p413 gws/ RR PHY413 GW Paper.pdf)

Questions: Online question session, can also email me!

## Lecture 1 <br> Shocks


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## Blast waves

After an explosion (e.g. supernova, atomic bomb) a fireball expands outwards

Explosion: energy E, at time $t=0$

## Blast waves

- In 1950, G. I. Taylor calculated how the fireball expansion relates to explosion energy and external density
- He reduced the problem to a self-similar scaling solution - basically using dimensional analysis.
- He reasoned that $r$ is a function of $E, \rho$ and $t$ :


## Blast waves

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## Blast waves

$$
r=C \rho^{x} E^{y} t^{z}
$$

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$$
m=\left[\mathrm{kg} \mathrm{~m}^{-3}\right]^{x}\left[\mathrm{~kg} \mathrm{~m}^{2} \mathrm{~s}^{-2}\right]^{y}[s]^{z}
$$

$$
m=\mathrm{kg}^{x+y} \mathrm{~m}^{-3 x+2 y} \mathrm{~s}^{-2 y+z}
$$

## Blast waves

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$$
m=\mathrm{kg}^{x+y} \mathrm{~m}^{-3 x+2 y} \mathrm{~s}^{-2 y+z}
$$

$0=x+y$
$1=-3 x+2 y$
$0=-2 y+z$
...mass
...length
...time

## Blast waves

$$
\begin{aligned}
& \stackrel{r}{\Longrightarrow}=C \rho^{x} E^{y} t^{z} \\
& \hline 0 \\
& 0=x+y \\
& 1=-3 x+2 y \\
& 0=-2 y+z \\
& \vec{x}=-1 / 5 \\
& y=1 / 5 \\
& z=2 / 5
\end{aligned}
$$

## Blast waves

$$
r=C \rho^{x} E^{y} t^{z}
$$

$\overrightarrow{0}=x+y$
$1=-3 x+2 y$
$0=-2 y+z$
$x=-1 / 5 \quad \therefore r=C \rho^{-1 / 5} E^{1 / 5} t^{2 / 5}$
$y=1 / 5$
$z=2 / 5$

## Blast waves

## $r=C \rho^{-1 / 5} E^{1 / 5} t^{2 / 5}$



- Taylor used high-speed photographs of small detonations in the lab to determine that $D \sim 1.033$ for a fireball expanding into air.
- The power of dimensional analysis is that even though the experiments were done on very small scales, we can be sure the scaling will still hold for explosions that are many, many orders of magnitude greater.
- Sedov later came up with a full solution (see: http:// www.mso.anu.edu.au/~geoff/AGD/Sedov.pdf), and so the above formula is usually referred to as the Taylor-Sedov solution.


## Blast waves

- Taylor used his equation and de-classified photos of the 1st atomic bomb tests to calculate the yield of the bomb.
$E \approx 1.033 \frac{r^{5} \rho}{t^{2}}$
$\rho \approx 1.1 \mathrm{~kg} \mathrm{~m}^{-3}$
$r \approx 140 \mathrm{~m}$
$t=25 \mathrm{~ms}$
$E \approx 9.778 \times 10^{13} \mathrm{~J}$
$E \approx 23.28$ kilotons


## Blast waves

- Taylor used his equation and de-classified photos of the 1st atomic bomb tests to calculate the yield of the bomb.
- The true (still classified in 1950) yield of the bomb was $\sim 18-20$ kilotons!!
$E \approx 23.28$ kilotons



## Blast waves

- Now calculate fluency of the Crab supernova of $E \approx \frac{r^{5} \rho}{t^{2}}$
$\rho \approx 10^{-21} \mathrm{~kg} \mathrm{~m}^{-3}$ $r \approx 3 \mathrm{pc} \sim 9 \times 10^{16} \mathrm{~m}$ $t=966 \mathrm{yrs} \sim 3 \times 10^{10}$

$$
E \approx 6.6 \times 10^{42} \mathrm{~J}
$$



## Blast waves

- What about the current speed of the blast wave?
$\dot{r} \approx(2 / 5) \rho^{-1 / 5} E^{1 / 5} t^{-3 / 5}$


## $\dot{r} \approx 1.2 \times 10^{6} \mathrm{~m} / \mathrm{s}$



## Blast waves

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 $\dot{r} \gg c_{s}$Blast wave is supersonic!


## Blast waves

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## $\dot{r} \approx 1.2 \times 10^{6} \mathrm{~m} / \mathrm{s}$

 $\dot{r} \gg c_{s}$Blast wave is supersonic! Shock!


## Shocks

- A shock occurs when a disturbance moves through a medium faster than the sound speed in the medium, i.e., sufficiently fast that a pressure wave cannot precede the disturbance.
- The conditions in the medium-temperature, density, bulk velocity -thus change almost instantaneously at the shock. The material which is hit by the shock receives no forewarning.



## Shocks

Derivation of strong shock 'jump conditions'

$\rho_{d}>\rho_{u}$

## Shocks

Switch to rest frame of the shock
'downstream'


Shocked gas particles appear to be moving away from the shock front


Un-shocked gas particles appear to be moving towards shock front.

$$
v_{d}<v_{u}
$$


(1) Mass conservation:

$$
\rho_{d} v_{d}=\rho_{u} v_{u}
$$

Mass per unit area flowing across shock front

(2) Momentum conservation:

Momentum per unit area flowing across shock front

$$
P_{d}+\rho_{d} v_{d}^{2}=P_{u}+\rho_{u} v_{u}^{2}
$$


(2) Momentum conservation:

Momentum per unit area flowing across shock front

$$
P_{d}+\rho_{d} v_{d}^{2}=P_{u}+\rho_{u} v_{u}^{2}
$$

Ram pressure (pressure from bulk motion)

(2) Momentum conservation:

Momentum per unit area flowing across shock front

Gas pressure (pressure from thermal motions)


Ram pressure (pressure from bulk motion)

## $\begin{array}{c:c:c}\text { 'downstream' } & \rho_{d} & v_{d} \\ & T_{d} & \rho_{u} \\ & & T_{u} \sim 0\end{array}$

(3) Energy conservation:

Energy per unit area flowing across shock front

## $\begin{array}{c:cc}\text { 'downstream' } & \rho_{d} & v_{d} \\ T_{d} & \rho_{u} & v_{u} \quad T_{u} \sim 0\end{array}$

(3) Energy conservation:

Energy per unit area flowing across shock front

$$
v_{d}\left[\frac{1}{2} \rho_{d} v_{d}^{2}+\rho_{d} \epsilon_{d}\right]+P_{d} v_{d}=v_{u}\left[\frac{1}{2} \rho_{u} v_{u}^{2}+\rho_{u} \epsilon_{u}\right]+P_{u} v_{u}
$$

$\varepsilon=$ internal energy per unit mass

(3) Energy conservation:

Energy per unit area flowing across shock front

energy / area / time
$\varepsilon=$ internal energy per unit mass

## 'downstream' <br> $$
T_{d}
$$

$\star \quad \rho_{d}$

Rankine-Hugoniot jump conditions:

1) $\rho_{d} v_{d}=\rho_{d} v_{u}$
2) $P_{d}+\rho_{d} v_{d}^{2}=P_{u}+\rho_{u} v_{u}^{2}$
3) $v_{d}\left[\frac{1}{2} \rho_{d} v_{d}^{2}+\rho_{d} \epsilon_{d}\right]+P_{d} v_{d}=v_{u}\left[\frac{1}{2} \rho_{u} v_{u}^{2}+\rho_{u} \epsilon_{u}\right]+P_{u} v_{u}$


Simplify (2):

$$
\begin{aligned}
& P_{d}+\rho_{d} v_{d}^{2}=P_{u}+\rho_{u} v_{u}^{2} \\
& T_{u} \sim 0 \Longrightarrow P_{u} \sim 0 \Longrightarrow \\
& P_{d}+\rho_{d} v_{d}^{2}=\rho_{u} v_{u}^{2}
\end{aligned}
$$

$$
\begin{align*}
\rho_{d} v_{d} & =\rho_{u} v_{u}  \tag{1}\\
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For ideal gas:
$\epsilon=\frac{3}{2} \frac{k T}{\bar{m}} \quad P=\frac{\rho}{\bar{m}} k T \quad \therefore \quad \rho \epsilon=\frac{3}{2} P$

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& \\
& v_{d}\left[\frac{1}{2} \rho_{d} v_{d}^{2}+\frac{3}{2} P_{d}\right]+P_{d} v_{d}=v_{u} \frac{1}{2} \rho_{u} v_{u}^{2}
\end{aligned}
$$

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$\#$

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(2) $\Longrightarrow \quad P_{d}=\rho_{u} v_{u}^{2}-\rho_{d} v_{d}^{2}$

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\end{array}
$$

$$
x-2
$$

$$
\Longrightarrow \quad \rho_{u} v_{u}^{3}+4 \rho_{d} v_{d}^{3}-5 \rho_{u} v_{u}^{2} v_{d}=0
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Sub in (1):

$$
\rho_{u} \psi_{u}^{\hat{3}}+4 \frac{\rho_{u}^{3}}{\rho_{d}^{2}} y_{u}^{/ 3}-5 \frac{\rho_{u}^{2}}{\rho_{d}} v_{u}^{\beta / \beta}=0
$$

$$
\begin{align*}
& \rho_{d} v_{d}=\rho_{u} v_{u}  \tag{1}\\
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$$
\rho_{u} \psi_{u}^{3}+4 \frac{\rho_{u}^{3}}{\rho_{d}^{2}} y \sqrt{3}-5 \frac{\rho_{u}^{2}}{\rho_{d}} v v_{u}^{\beta}=0
$$

$$
\times \frac{\rho_{d}^{2}}{\rho_{u}^{3}} \Longrightarrow\left(\frac{\rho_{d}}{\rho_{u}}\right)^{2}+4-5 \frac{\rho_{d}}{\rho_{u}}=0
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$$
\times \frac{\rho_{d}^{2}}{\rho_{u}^{3}} \Longrightarrow\left(\frac{\rho_{d}}{\rho_{u}}\right)^{2}+4-5 \frac{\rho_{d}}{\rho_{u}}=0
$$

$$
\left(\frac{\rho_{d}}{\rho_{u}}-1\right)\left(\frac{\rho_{d}}{\rho_{u}}-4\right)=0
$$

Trivial: $v_{d}=v_{u} ; \quad \rho_{d}=\rho_{u}$

$$
\begin{equation*}
\rho_{d} v_{d}=\rho_{u} v_{u} \tag{1}
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$\times \frac{\rho_{d}^{2}}{\rho_{u}^{3}} \Longrightarrow\left(\frac{\rho_{d}}{\rho_{u}}\right)^{2}+4-5 \frac{\rho_{d}}{\rho_{u}}=0$

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\end{align*}
$$

Strong shock jump conditions:

## $\underline{\rho_{d}}=4$ <br> $\rho_{u}$

$\underline{v_{u}}=4$
$v_{d}$

## Shocks

## Back in the rest frame of the ISM

Assumptions:

- Ideal gas, T.E./m=(3/2)kT
- Mach number: $\mathscr{M} \equiv v / c_{s} \gg 1$
- $\quad T_{u} \sim 0$
- Ignored viscosity (only important within mean free path from shock front)



## Compact object accretion

Compact object: black hole / neutron
star / white dwarf

Companion star

## Compact object accretion

- Compact object = black hole or neutron star: system called an X-ray binary
- Compact object = white dwarf: system called cataclysmic variable or dwarf nova (historic)
- White dwarf systems: WD always observed to be more massive than companion
- High mass X-ray binaries (HMXBs):

$$
M_{\text {companion }}>M_{\mathrm{co}}
$$

- Low mass X-ray binaries (LMXBs):

$$
M_{\text {companion }}<M_{\mathrm{co}}
$$

## Compact object accretion LMXBs: Roche-Lobe Overflow

- Roche potential: companion star (CS) occupies equipotential surface.
- CS can fill its Roche Lobe if it swells during its evolution, or binary separation reduces due to angular momentum loss due to e.g. stellar wind mass loss or gravitational waves.
- Then material passes through $L_{1}$ (inner Lagrange point).
- In-falling material forms an accretion disc



## Compact object accretion HMXBs: Stellar Wind Capture

- $M \gtrsim 15 M_{\odot}$ stars can have a strong stellar wind.
- Wind can be captured by the compact object.
- Accretion luminosity therefore depends on mass outflow rate of wind.
- Can also get Roche-Lobe overflow in HMXBs, but less common
- Intermediate case: Cygnus X-1 has a focused wind



## Compact object accretion

 Magnetic fields- B-field of NS / WD can interrupt disc and channel material directly to magnetic poles.

CV (white dwarf) / X-ray pulsar (neutron star)


## Compact object accretion

## Magnetic fields

- B-field of NS / WD can interrupt disc and channel material directly to magnetic poles.
- For strongest field WDs, there can be no disc at all.


## Polar



## Compact object accretion

Magnetic fields

- Gravitational free fall in accretion column
- Shock as material is halted
- Homework problem: calculate $\mathrm{T}_{\mathrm{d}}$ (hot enough to emit hard $X$ rays)
$T_{u} \sim 0$


