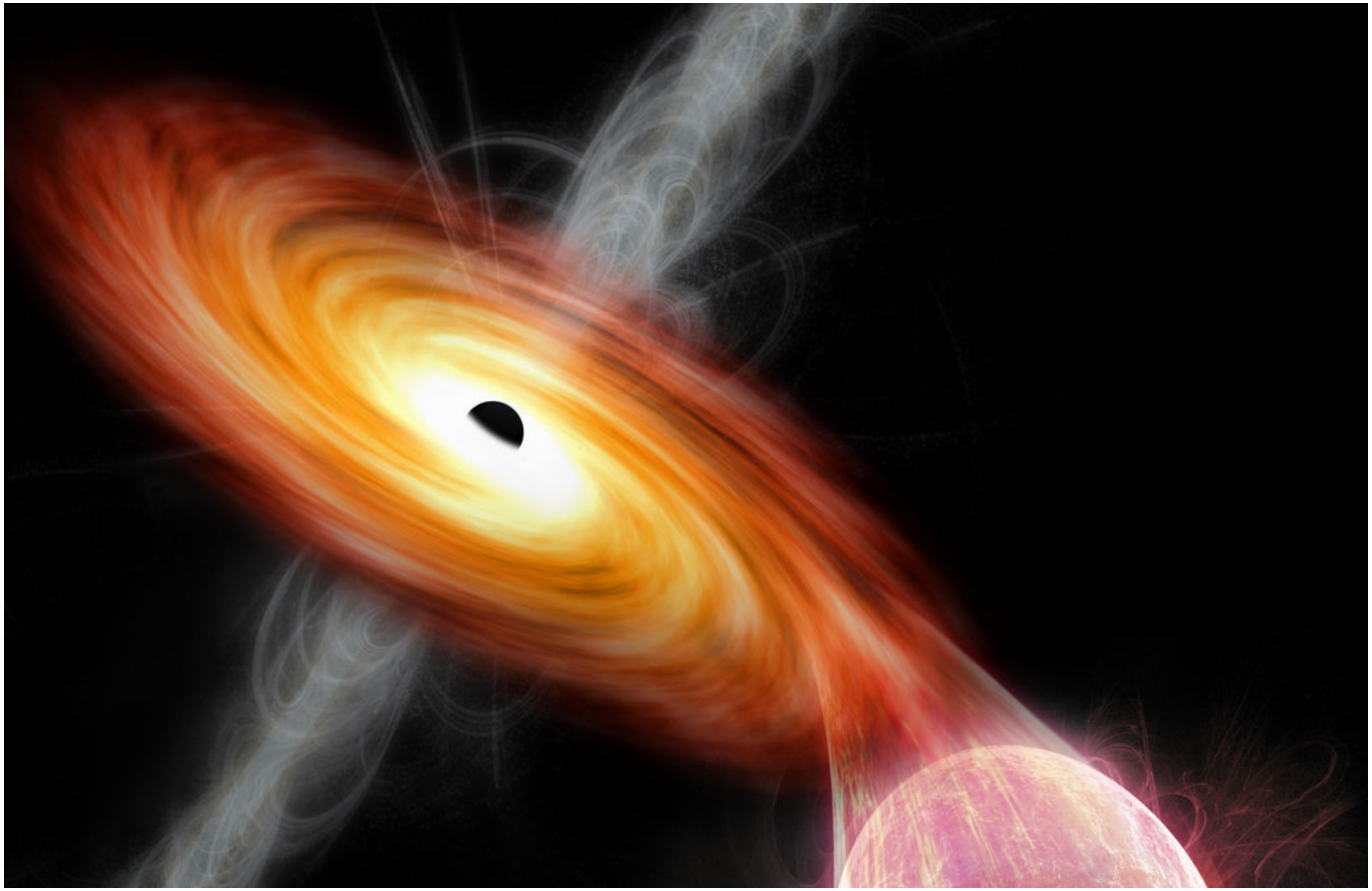
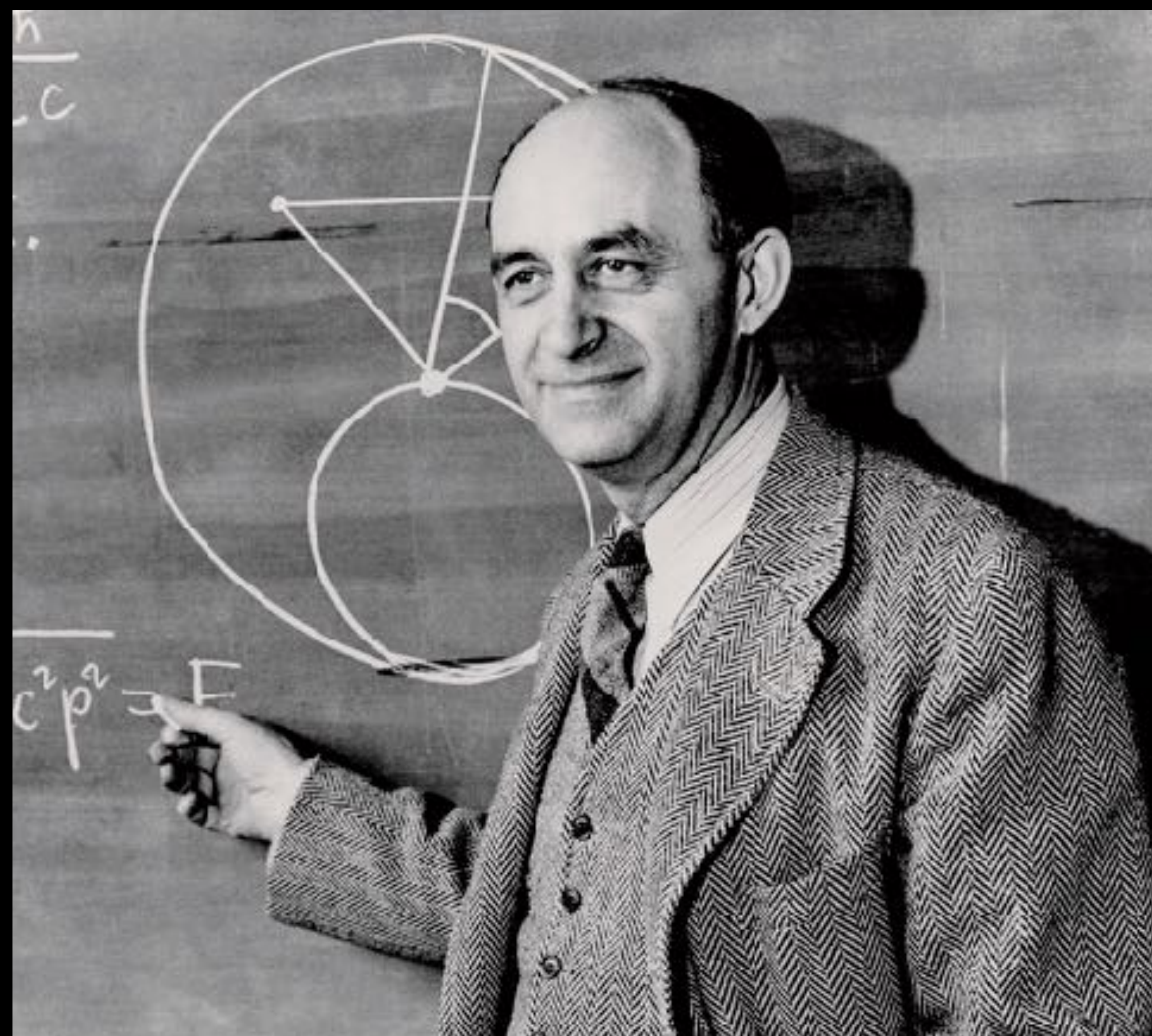
High Energy Astrophysics

Dr. Adam Ingram



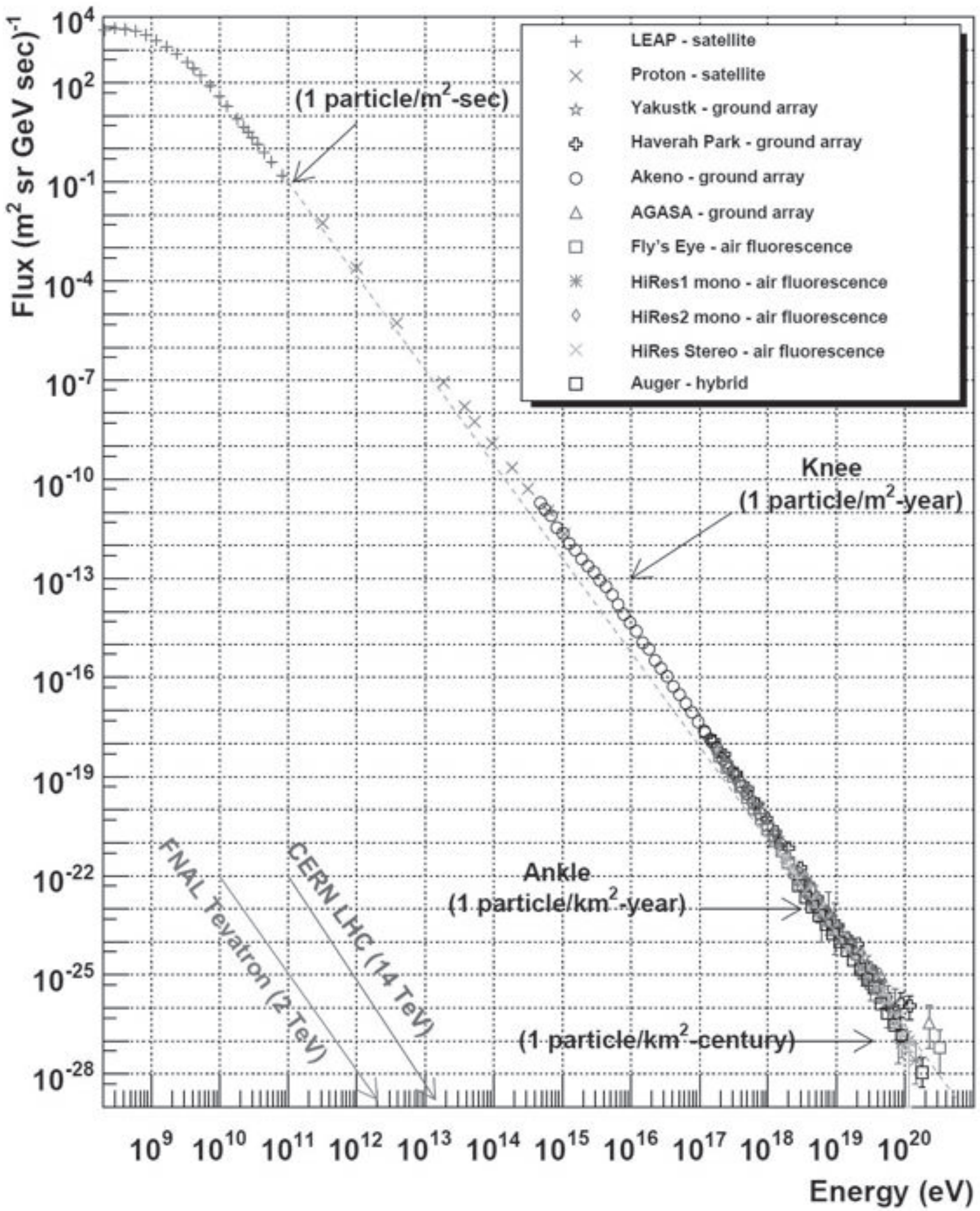
Lecture 2
Shock Acceleration



Enrico Fermi

Cosmic Rays

Earth is hit by highly energetic elementary particles



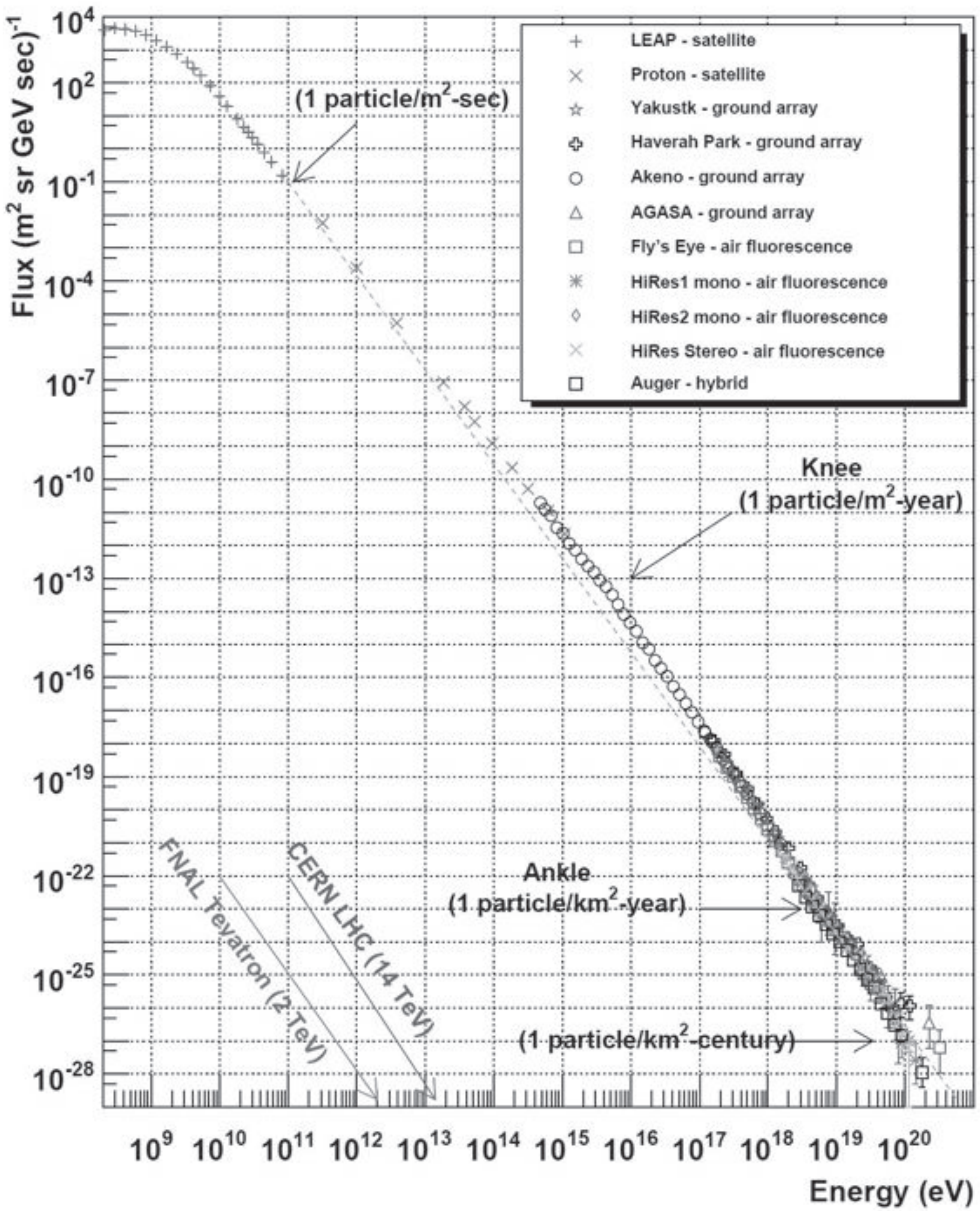
- Discovered in 1912 by Hess on a balloon flight (1936 Nobel prize)
- ~85% protons, ~12% He nuclei, ~1% heavier nuclei, ~2% electrons.
- ~power-law across many decades in energy:

$$\frac{dN}{dE} \propto E^{-k}, \quad k \sim 2.5$$

- "Ankle and "knee"
- Highest energies ~10^{20} eV!!

Cosmic Rays

Earth is hit by highly energetic elementary particles



- Discovered in 1912 by Hess on a balloon flight (1936 Nobel prize)
- ~85% protons, ~12% He nuclei, ~1% heavier nuclei, ~2% electrons.
- ~power-law across many decades in energy:

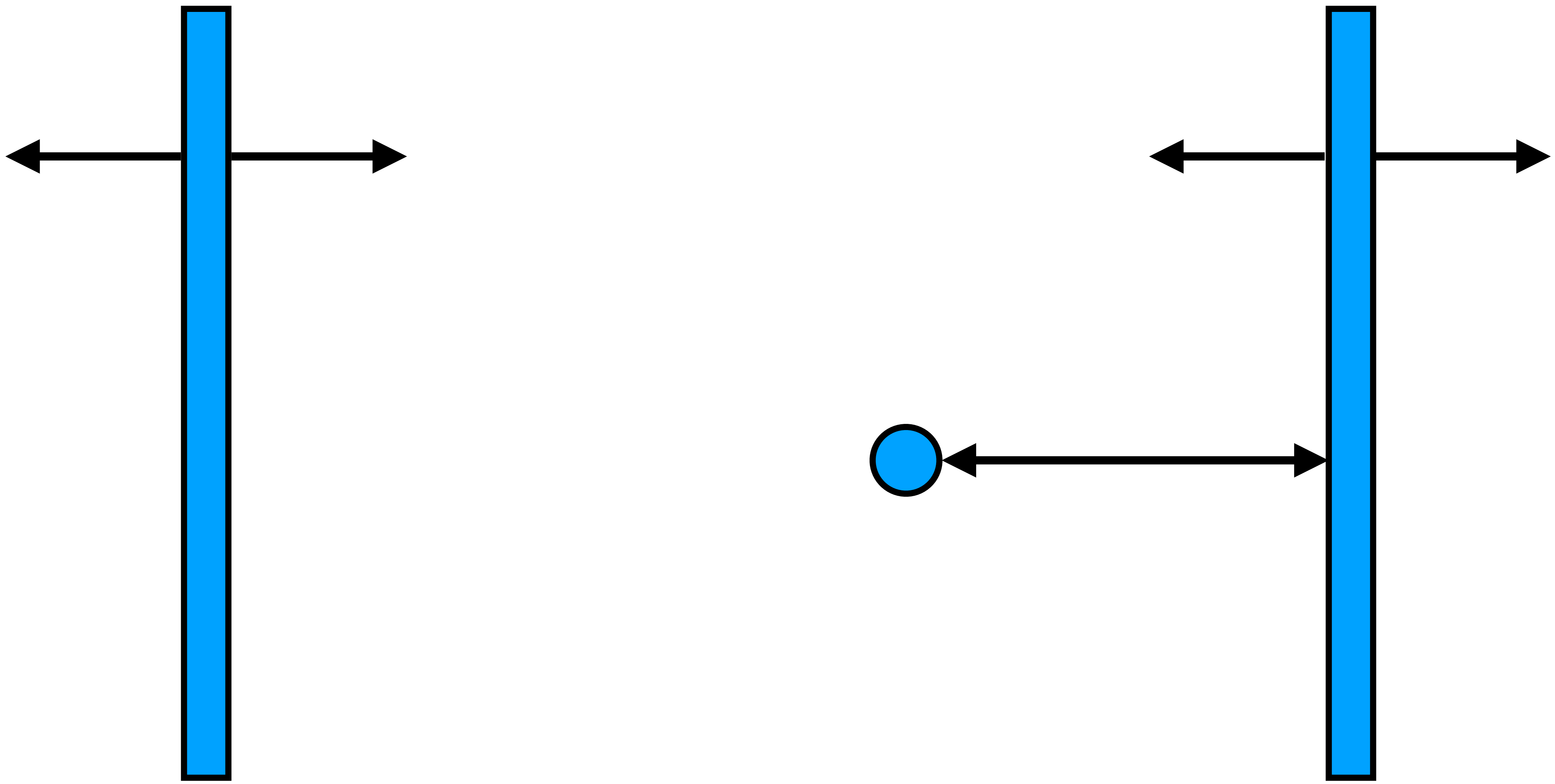
$$\frac{dN}{dE} \propto E^{-k}, \quad k \sim 2.5$$

- "Ankle and "knee"
- Highest energies ~10^{20} eV!!

How to accelerate particles to such high energies?

Fermi acceleration

Fermi (1949) considered mildly relativistic particles reflecting off randomly moving “magnetic mirrors” in the Galaxy.

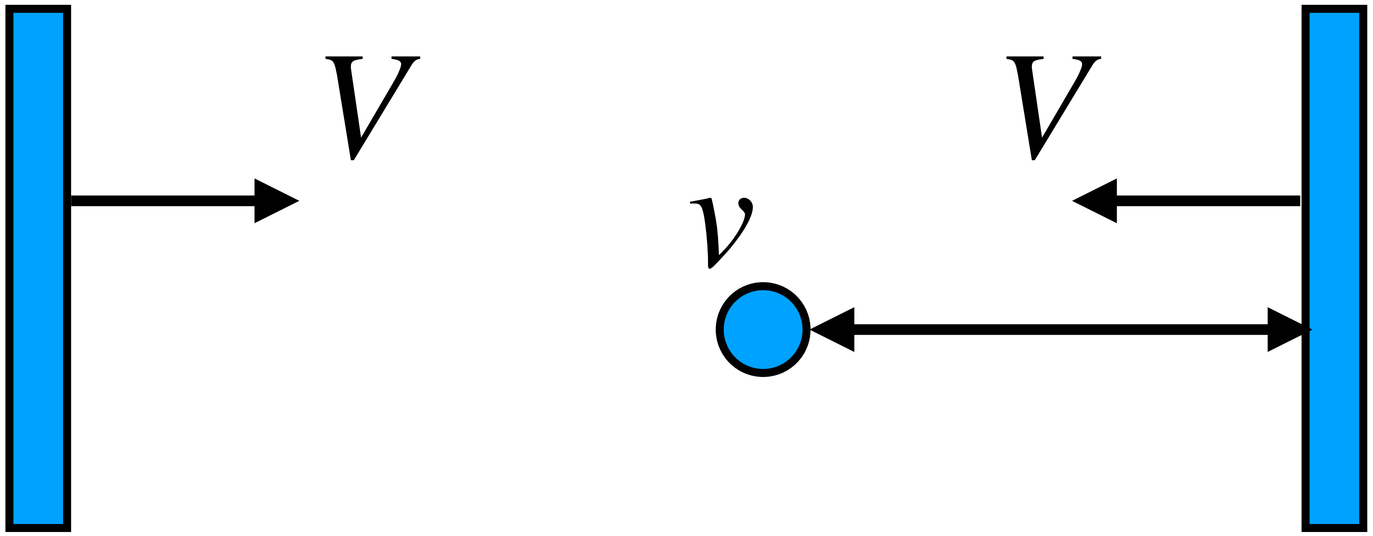


He showed that particles can, on average gain energy from bouncing off these mirrors many times.

Fermi acceleration

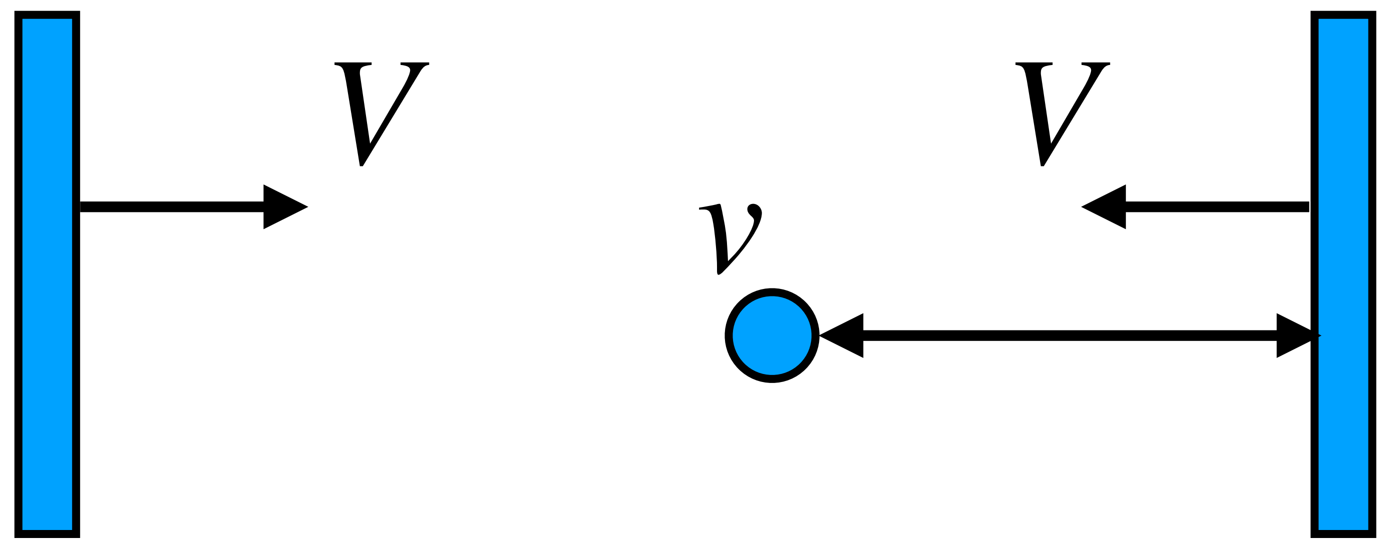
We can understand the concept by thinking of the mirrors moving together like the ‘trash compactor’ scene in Star Wars.

Fermi acceleration



Assume: $V \ll c$ $\quad v \sim c$ $\quad$ No recoil of mirrors

Fermi acceleration

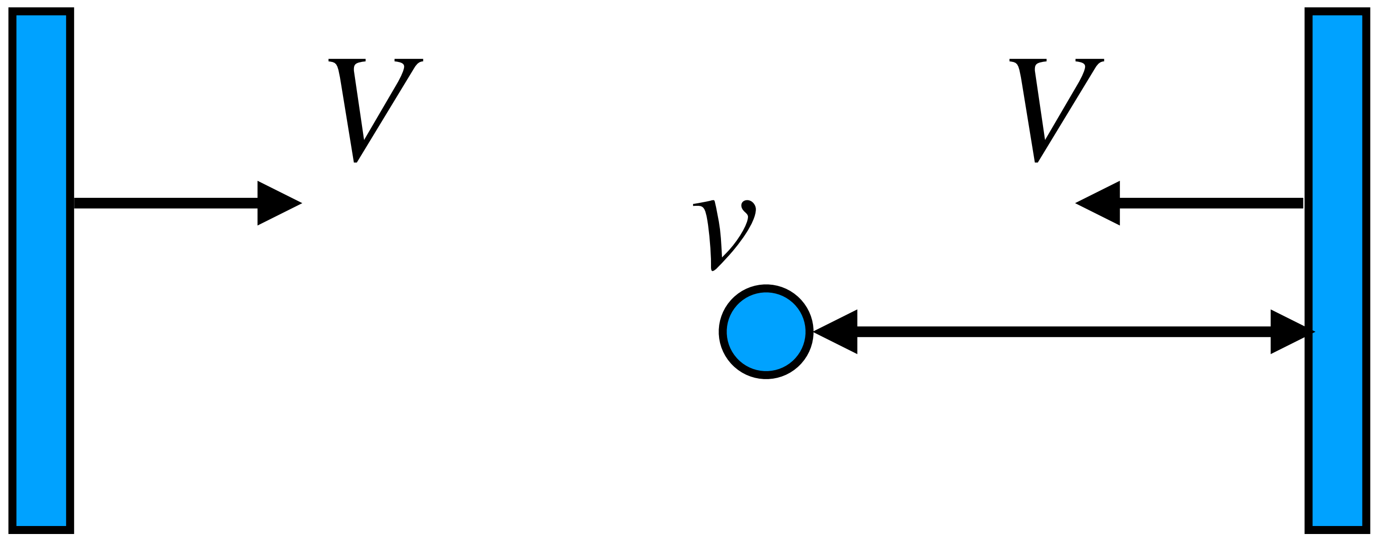


Assume: $V \ll c$ $v \sim c$ No recoil of mirrors

Momentum increase per collision $= \gamma m V$

$$\gamma = \frac{1}{\sqrt{1 - (V/c)^2}}$$

Fermi acceleration



Assume: $V \ll c$ $\quad v \sim c$ $\quad$ No recoil of mirrors

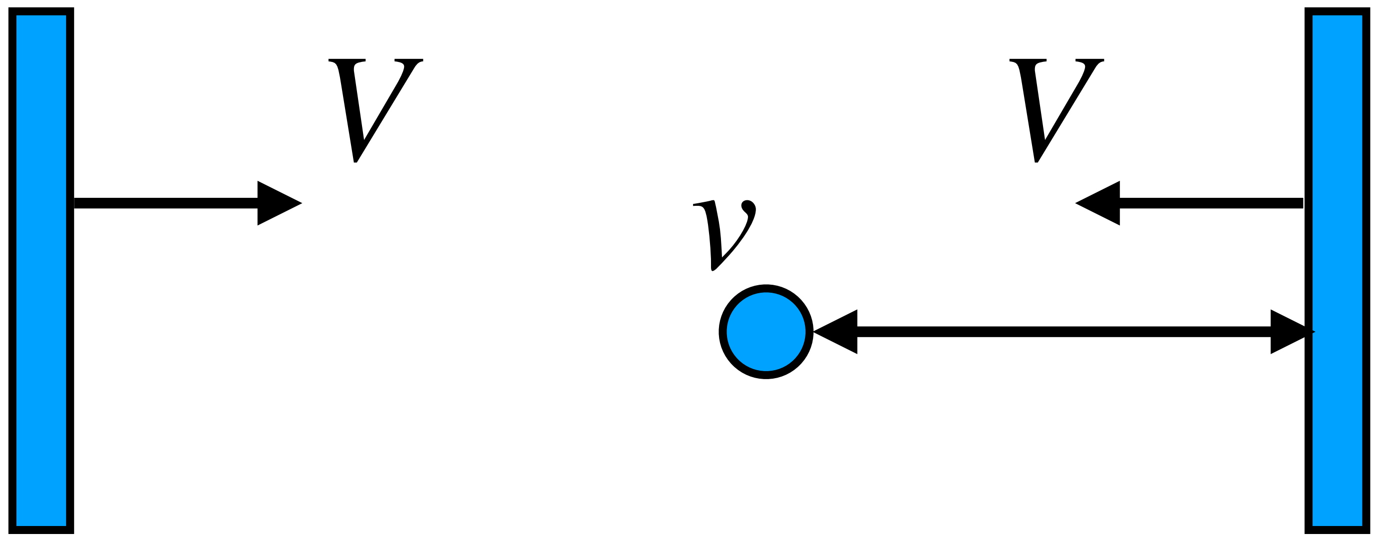
---

Momentum increase per collision $= \gamma m V$

$$\gamma = \frac{1}{\sqrt{1 - (V/c)^2}}$$

$$E^2 = (pc)^2 + (m_o c^2)^2$$

Fermi acceleration



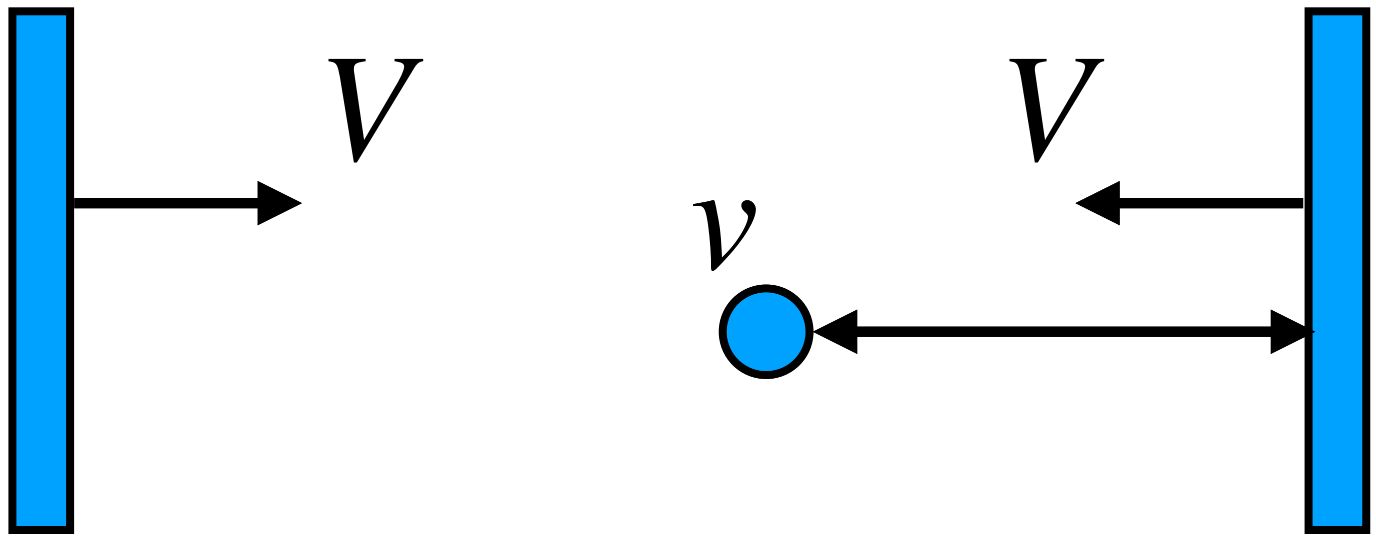
Assume: $V \ll c$ $v \sim c$ No recoil of mirrors

Momentum increase per collision $= \gamma m V$ $\gamma = \dfrac{1}{\sqrt{1-(V/c)^2}}$

$$E^2 = (pc)^2 + (m_o c^2)^2$$

Energy increase per collision $\approx \gamma m V c$

Fermi acceleration



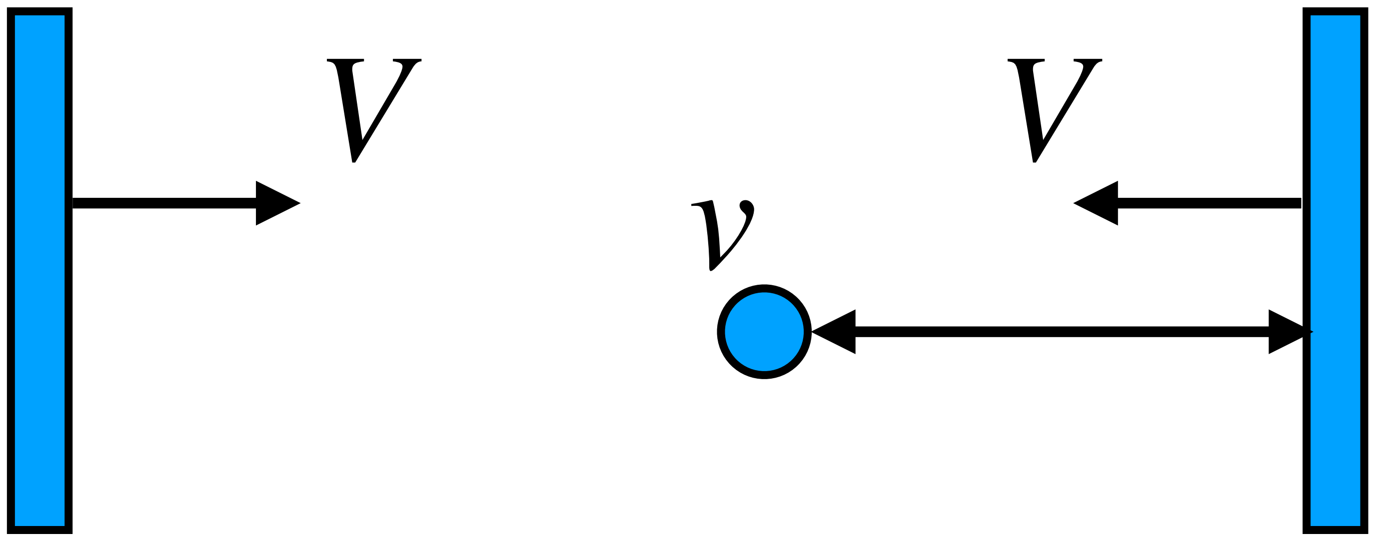
Assume: $V \ll c$ $v \sim c$ No recoil of mirrors

Momentum increase per collision $= \gamma m V$ $\gamma = \dfrac{1}{\sqrt{1 - (V/c)^2}}$

$$E^2 = (pc)^2 + (m_o c^2)^2$$

$$\text{Energy increase per collision} \approx \gamma m V c = \gamma m c^2 \frac{V}{c}$$

Fermi acceleration



Assume: $V \ll c$ $v \sim c$ No recoil of mirrors

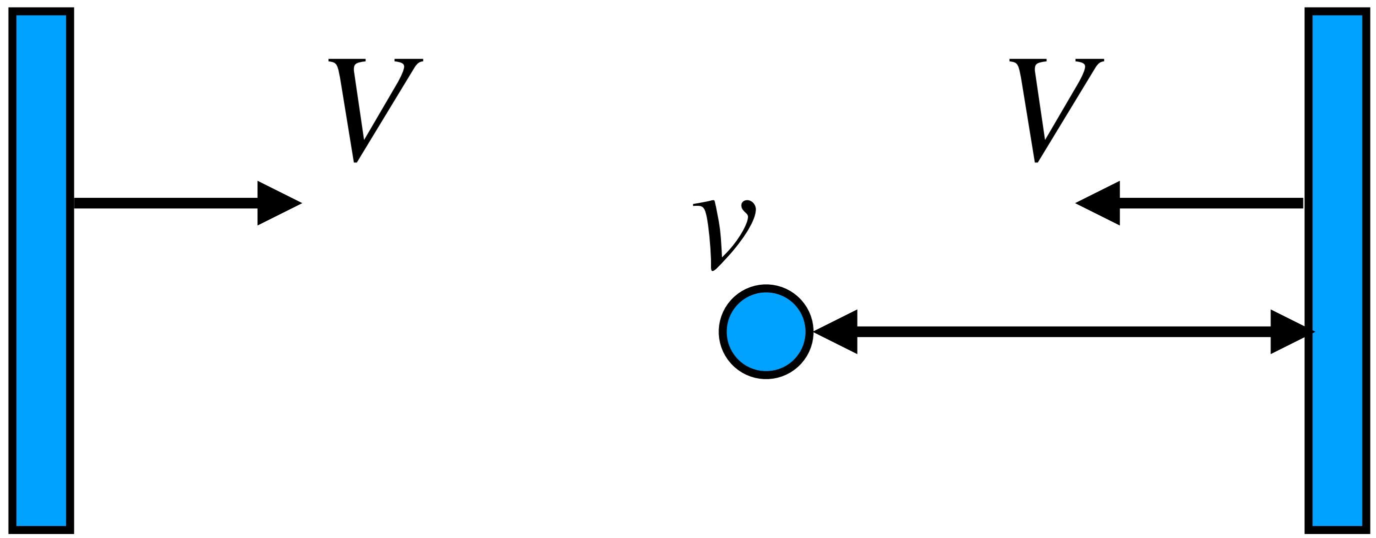
Momentum increase per collision $= \gamma m V$

$$\gamma = \frac{1}{\sqrt{1 - (V/c)^2}}$$

$$E^2 = (pc)^2 + (m_o c^2)^2$$

$$\text{Energy increase per collision} \approx \gamma m V c = \gamma m c^2 \frac{V}{c} = \frac{V}{c} E$$

Fermi acceleration



Assume: $V \ll c$ $\quad v \sim c \quad$ No recoil of mirrors

---

Momentum increase per collision $= \gamma m V$ $\qquad \gamma = \dfrac{1}{\sqrt{1-(V/c)^2}}$

$$E^2 = (pc)^2 + (m_o c^2)^2$$

$$\text{Energy increase per collision} \approx \gamma m V c = \gamma m c^2 \frac{V}{c} = \frac{V}{c} E$$

$$\therefore\ \beta \equiv \frac{\text{Energy after collision}}{\text{Energy before collision}} = 1 + \frac{\Delta E}{E} \approx 1 + \frac{V}{c}$$

Fermi acceleration

- Same fractional energy gain per crossing
- Energy after n crossings: $E = E_0 \beta^n$

Fermi acceleration

- Same fractional energy gain per crossing
- Energy after n crossings: $E = E_0\beta^n$
- P = probability of particle remaining in accelerator after each crossing. Particles remaining after n crossings: $N = N_0P^n$

Fermi acceleration

- Same fractional energy gain per crossing
- Energy after n crossings: $E = E_0 \beta^n$
- P = probability of particle remaining in accelerator after each crossing. Particles remaining after n crossings: $N = N_0 P^n$
- Eliminate n:

$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{n \ln(P)}{n \ln(\beta)}$$

Fermi acceleration

- Same fractional energy gain per crossing
- Energy after n crossings: $E = E_0 \beta^n$
- P = probability of particle remaining in accelerator after each crossing. Particles remaining after n crossings: $N = N_0 P^n$
- Eliminate n:

$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{n \ln(P)}{n \ln(\beta)}$$

$$\therefore N = N_0 (E/E_0)^{\ln(P)/\ln(\beta)}$$

Fermi acceleration

- Same fractional energy gain per crossing
- Energy after n crossings: $E = E_0 \beta^n$
- P = probability of particle remaining in accelerator after each crossing. Particles remaining after n crossings: $N = N_0 P^n$
- Eliminate n:

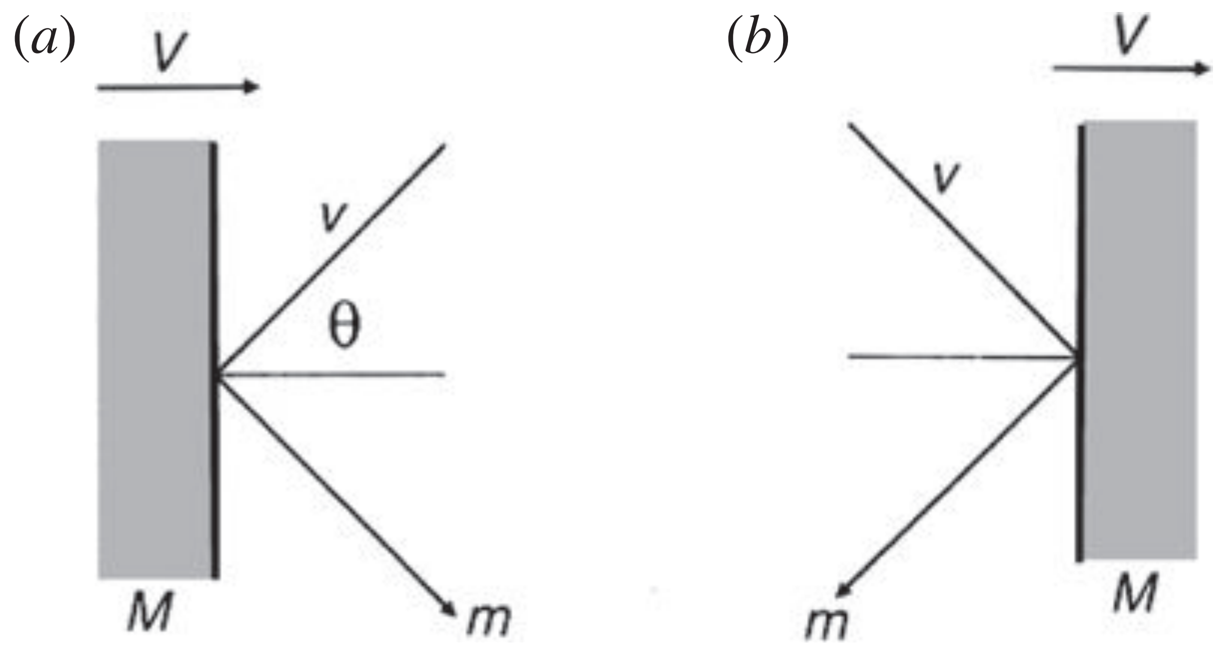
$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{n \ln(P)}{n \ln(\beta)}$$

$$\therefore N = N_0 (E/E_0)^{\ln(P)/\ln(\beta)}$$

$$\therefore \frac{dN}{dE} \propto E^{\ln(P)/\ln(\beta) - 1}$$

…get a power-law spectrum!

Fermi acceleration



From Longair (1994)

Illustrating the collision between a particle of mass m and a cloud of mass M: (a) a head-on collision; (b) a following collision. The probabilities of head-on and following collisions are proportional to the relative velocities of approach of the particle and the cloud, namely, $v + V\cos\theta$ for (a) and $v - V\cos\theta$ for (b). Since $v \approx c$, the probabilities are proportional to $1 + (V/c)\cos\theta$ where $0 < \theta < \pi$.

Fermi imagined randomly moving magnetic mirrors, and particles moving at all angles to the mirrors. Collisions with approaching mirrors more likely than those with receding mirrors, so net energy gain. Net gain per collision is $\Delta E = (8/3)(V/c)^2 E$ (see e.g. Longair 1994 for derivation). For this reason, Fermi's original model is called *second order Fermi acceleration*.

Fermi acceleration

Problems

1. How to get particles travelling at mildly relativistic v in the first place?
2. Random velocities of Galactic clouds is $V/c \sim 10^{-4}$, so fractional energy gain per collision is $\sim \Delta E/E \sim 10^{-8}$. Mean free path between clouds is ~0.1 pc, so very slow energy gain!
3. Get a power-law spectrum, but why specifically $\sim E^{-2.5}$?

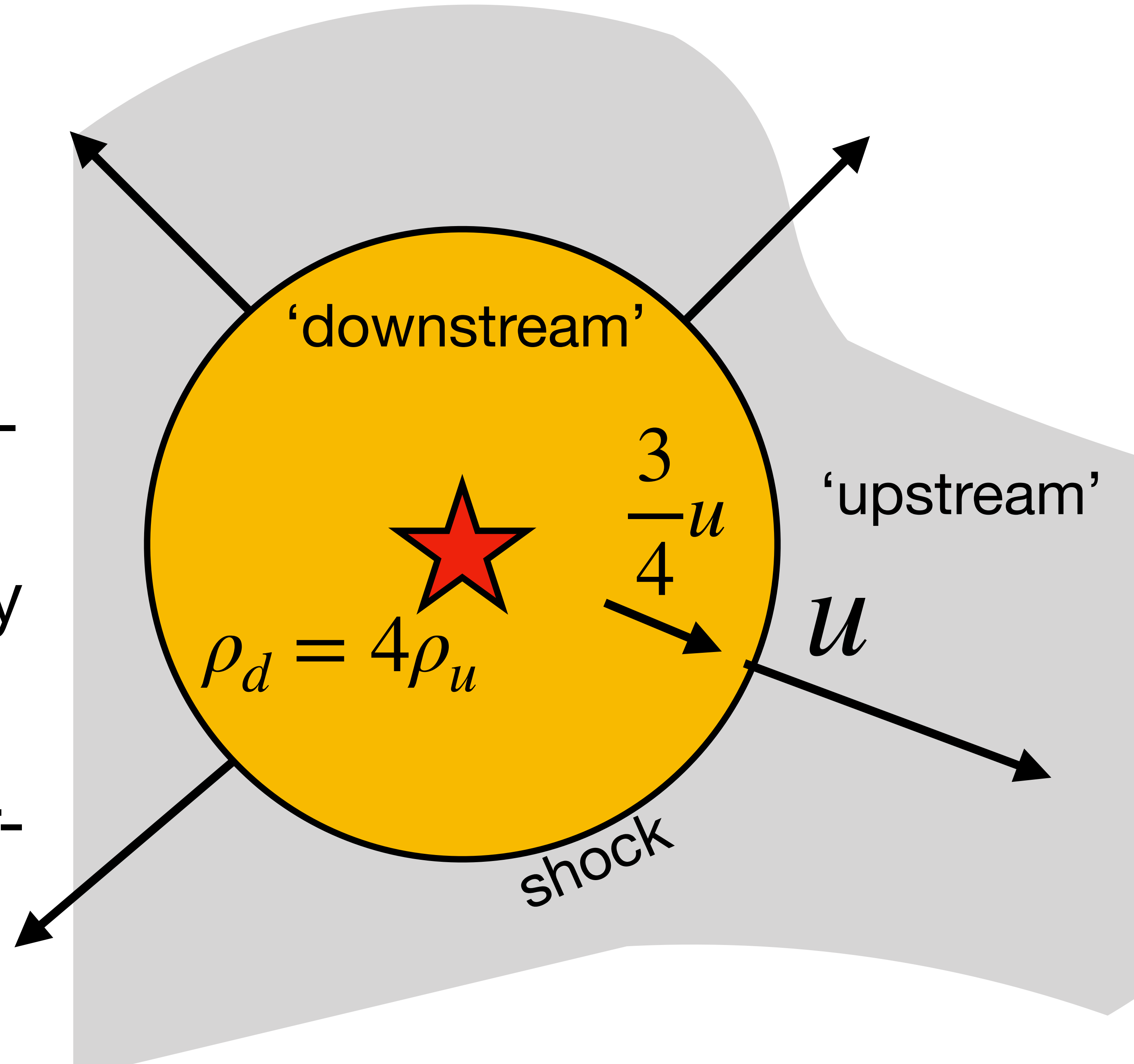
Shock acceleration

Considering particles at a shock front solves all the problems:

1. Already some mildly relativistic particles from the supernova / black hole jet causing the shock wave;

2. Mechanism to only get head-on collisions so ΔE/E~V/c, therefore easier to get to very high energies;

3. Can predict a specific power-law index.



Bell (1978)

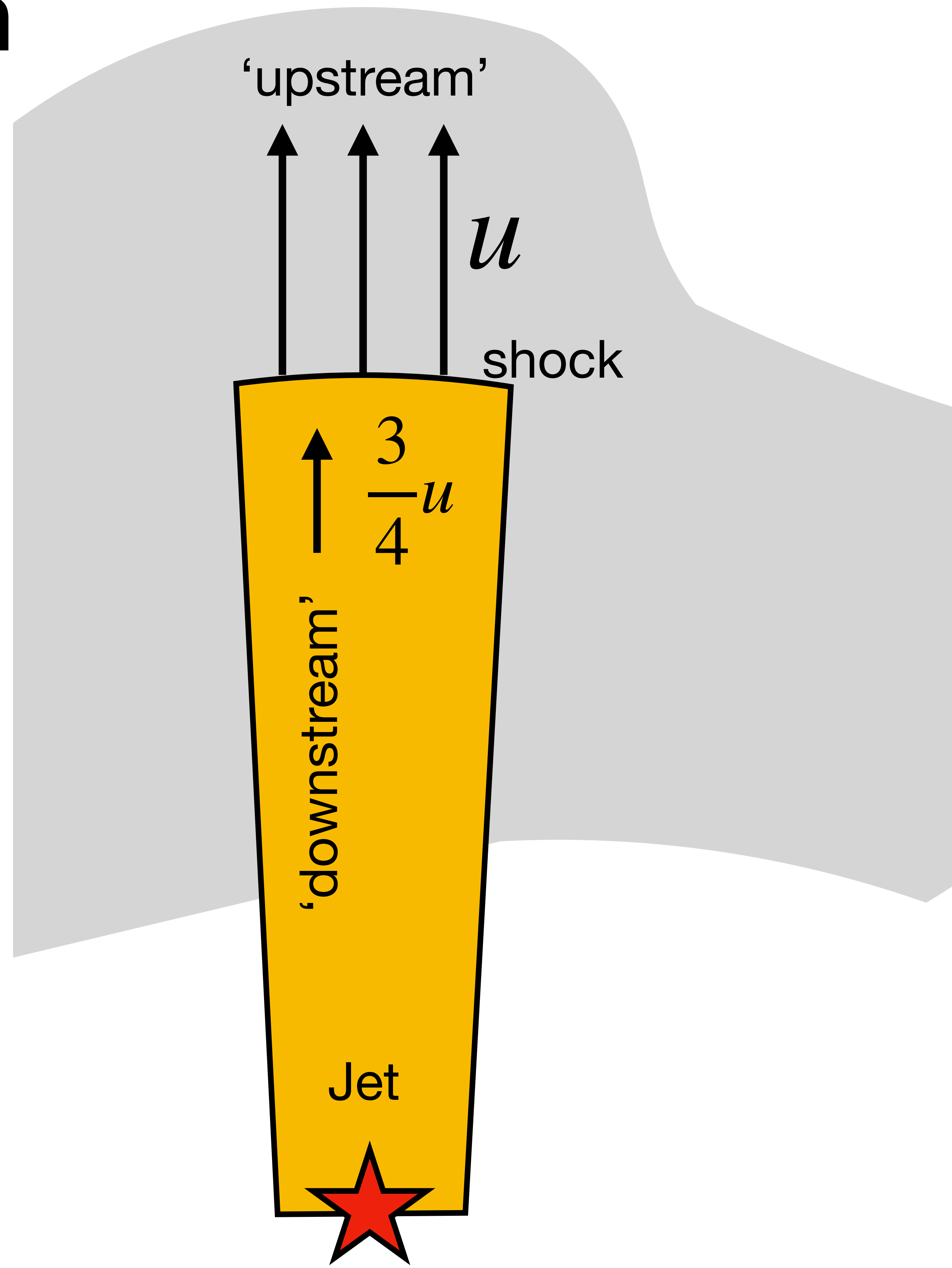
Shock acceleration

Considering particles at a shock front solves all the problems:

1. Already some mildly relativistic particles from the supernova / black hole jet causing the shock wave;

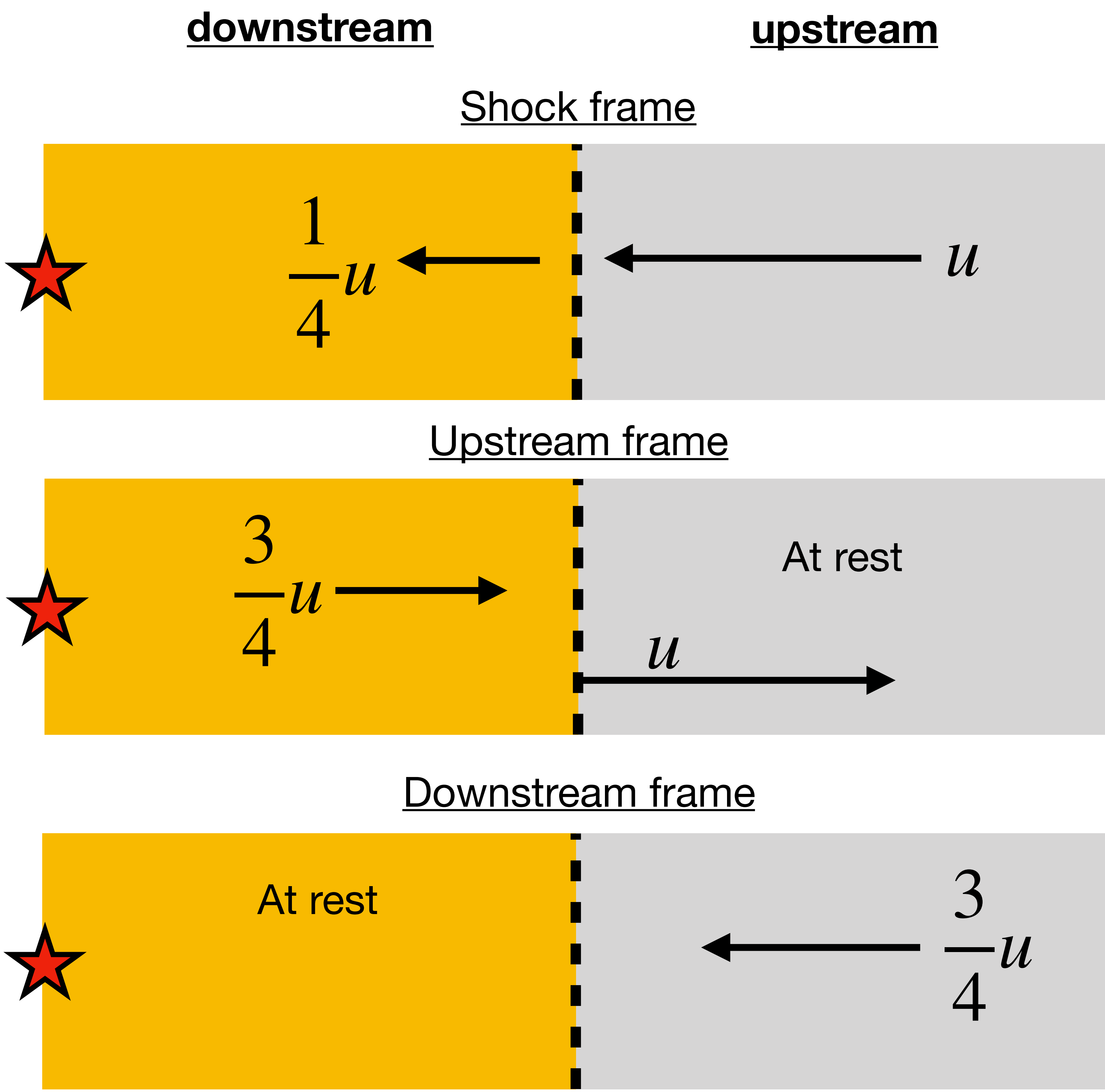
2. Mechanism to only get head-on collisions so ΔE/E~V/c, therefore easier to get to very high energies;

3. Can predict a specific power-law index.

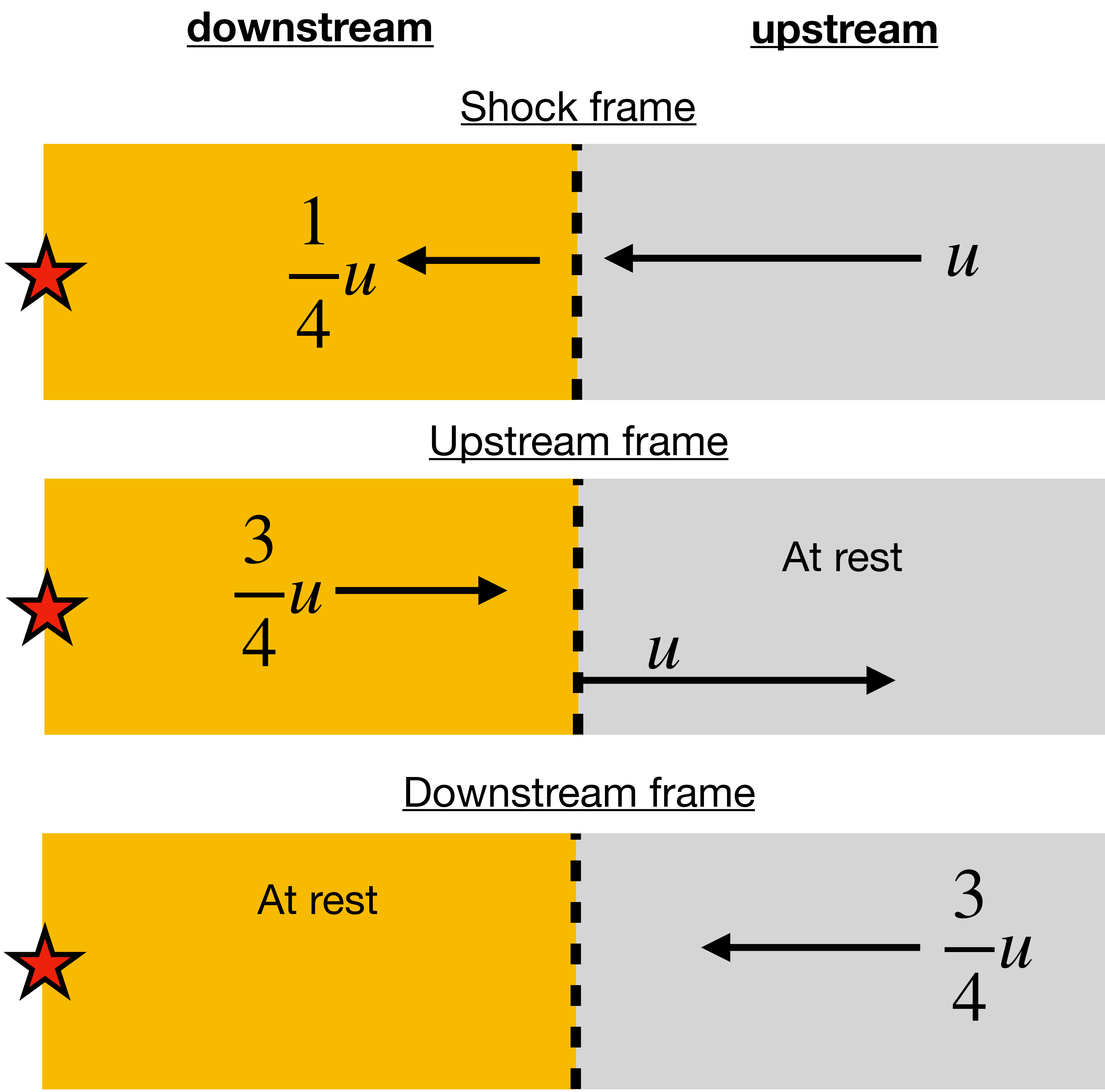


Bell (1978)

Shock acceleration

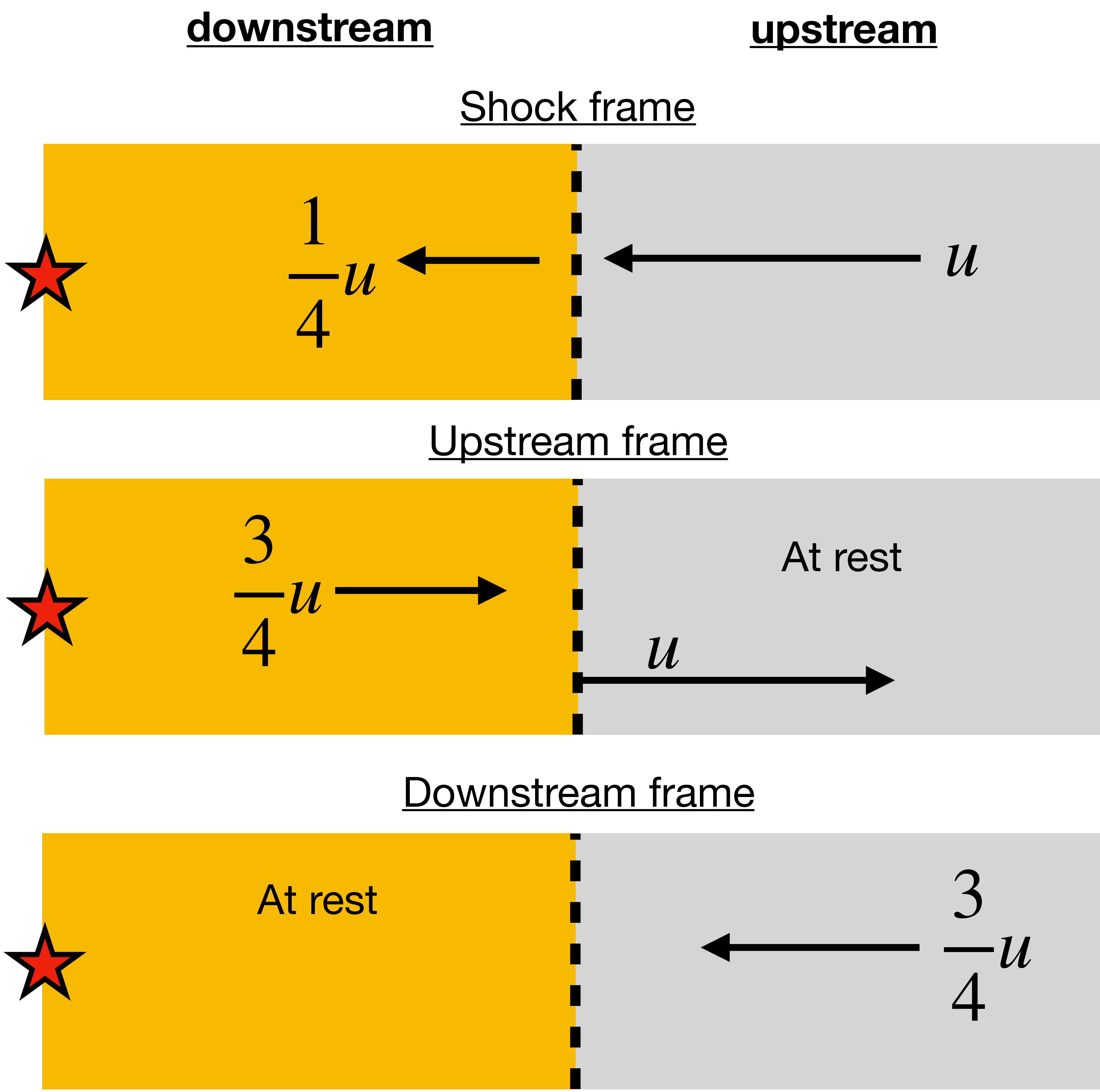


Shock acceleration



Upstream particles see downstream particles moving towards them at V=(3/4)u

Shock acceleration



Upstream particles see downstream particles moving towards them at V=(3/4)u

Downstream particles see upstream particles moving towards them at V=(3/4)u

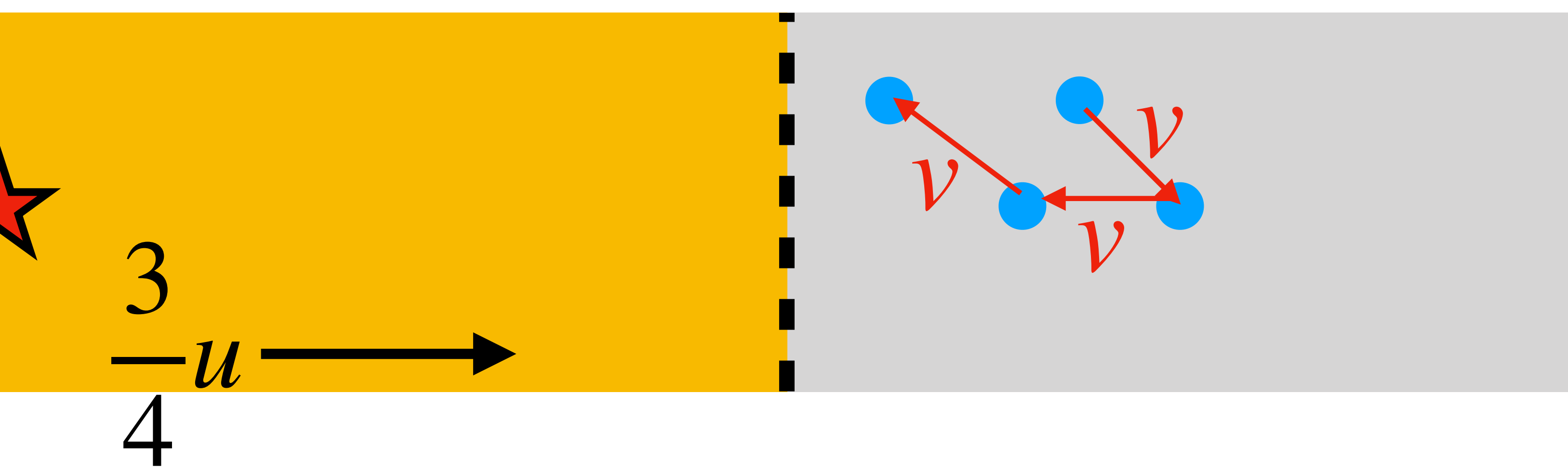
Shock acceleration

downstream **upstream**

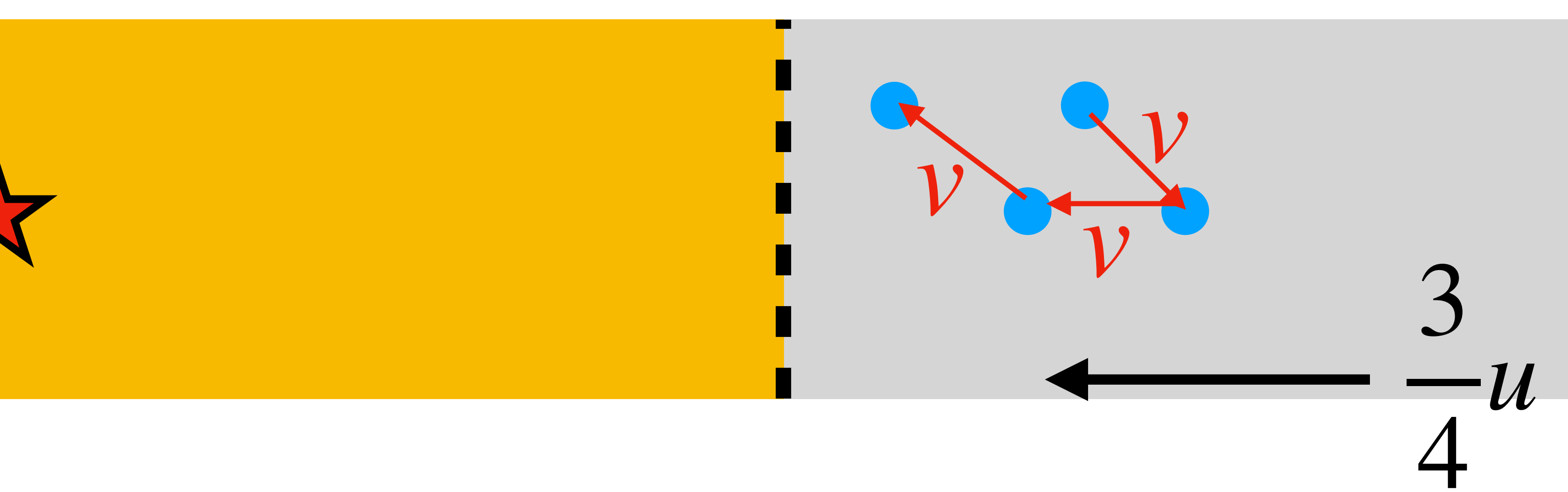
$$u << c$$

$$v \sim c$$

Upstream frame



Downstream frame



- Particles scattering on upstream side with isotropic velocity distribution in upstream rest frame.

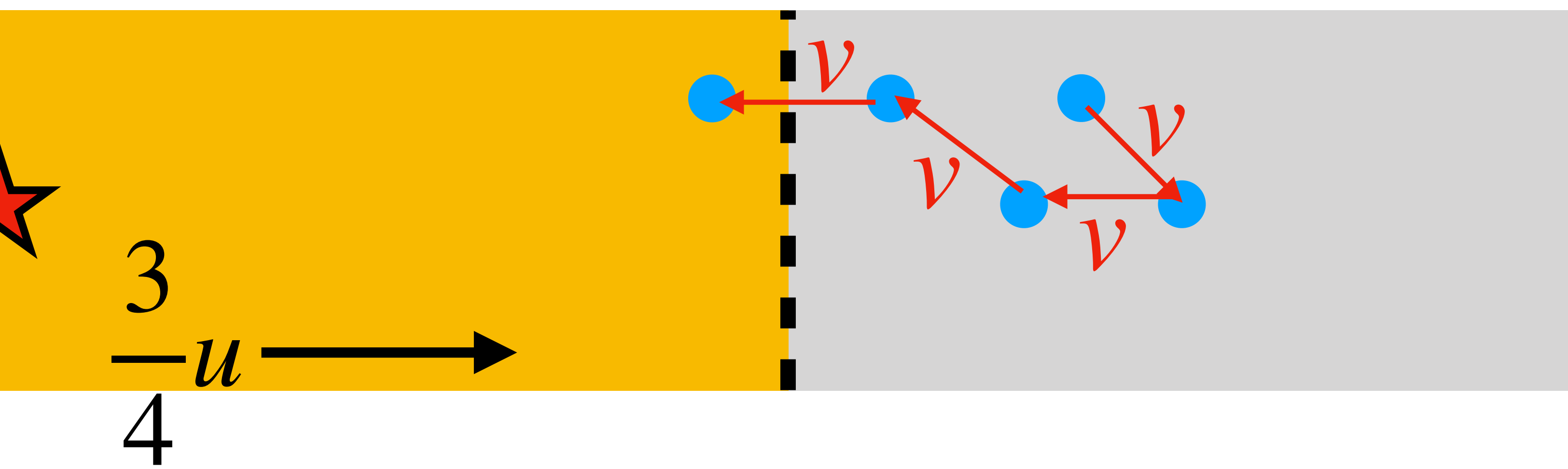
Shock acceleration

downstream **upstream**

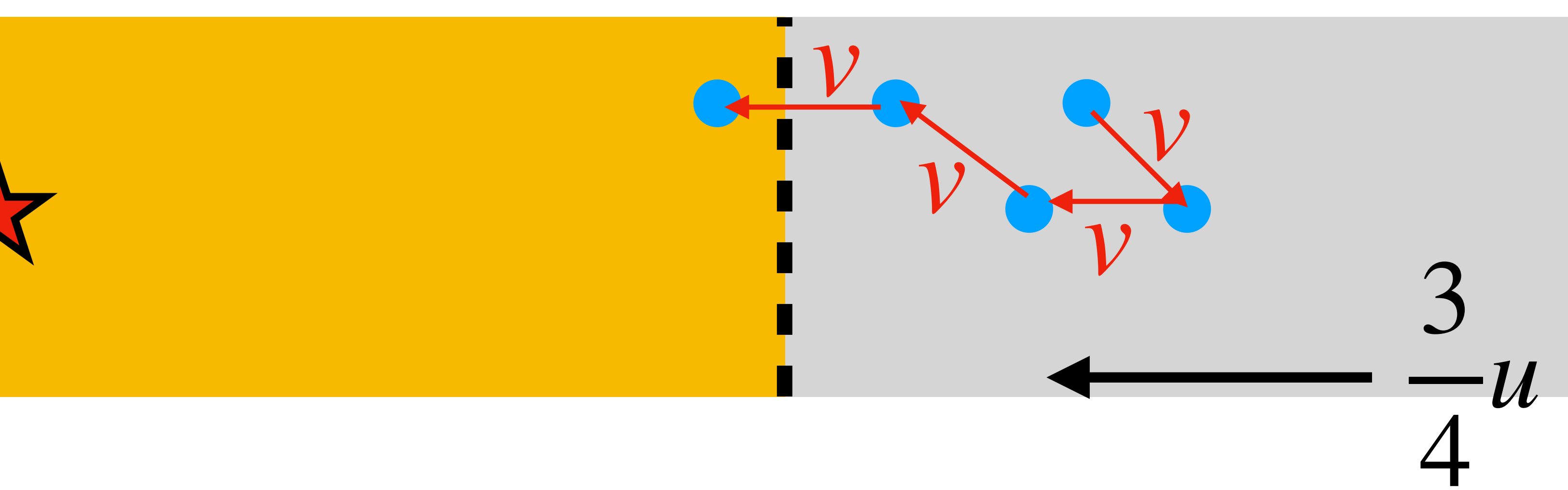
$$u << c$$

$$v \sim c$$

Upstream frame



Downstream frame



- Particles scattering on upstream side with isotropic velocity distribution in upstream rest frame.
- Eventually cross shock front.

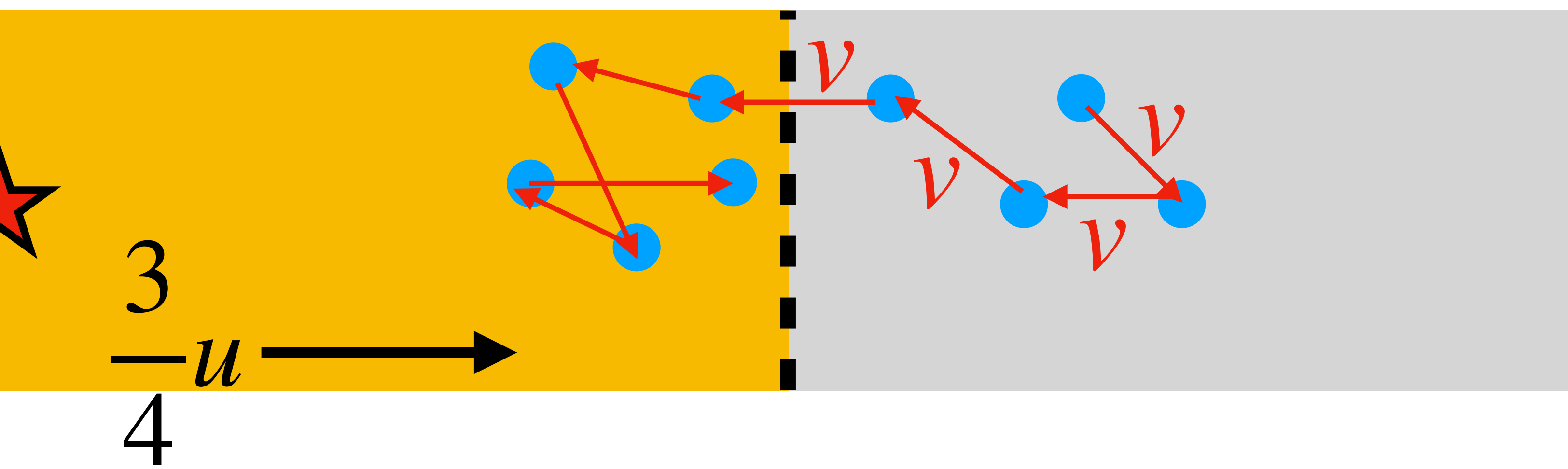
Shock acceleration

downstream **upstream**

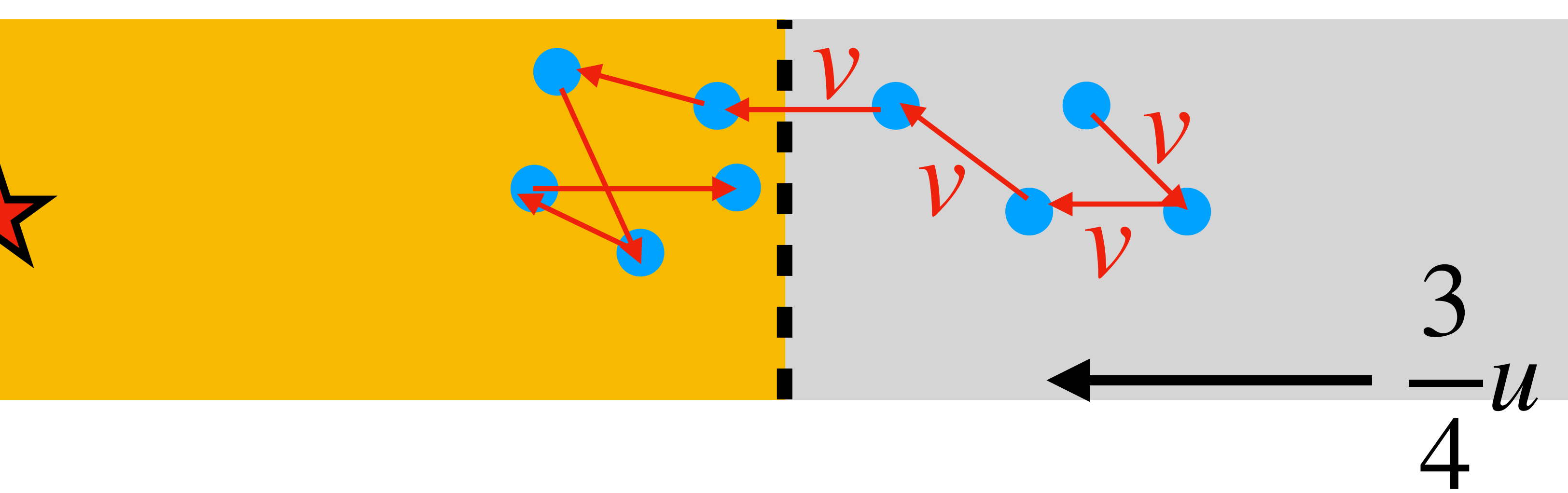
$$u << c$$

$$v \sim c$$

Upstream frame



Downstream frame



- Particles scattering on upstream side with isotropic velocity distribution in upstream rest frame.
- Eventually cross shock front.
- More scatterings on downstream side. Eventually velocity distribution becomes isotropic in *downstream* rest frame.

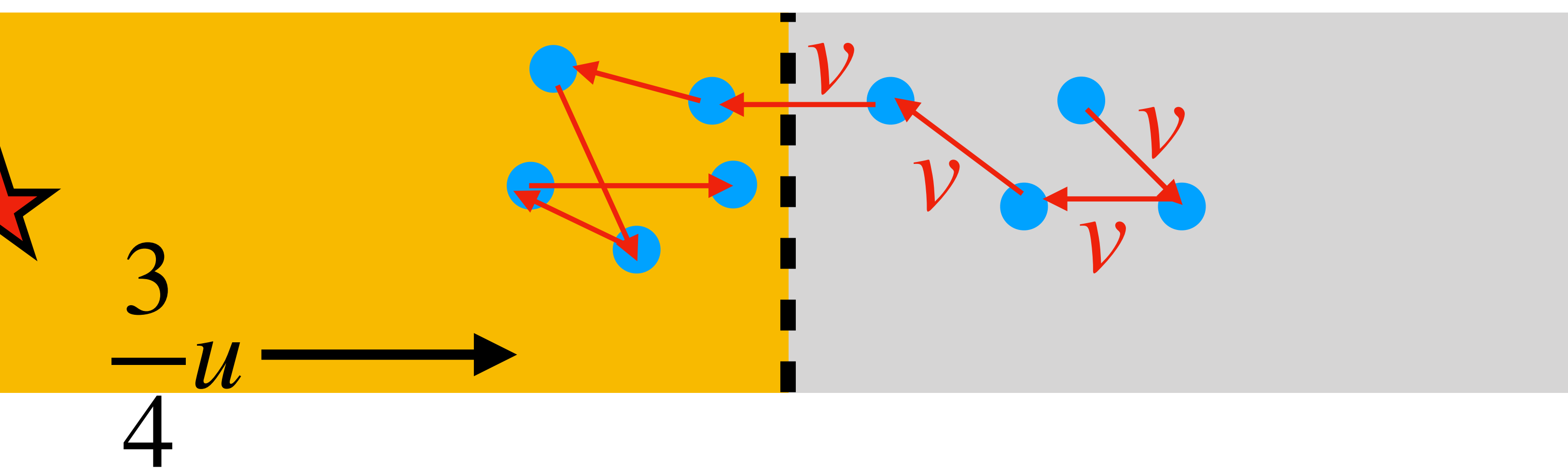
Shock acceleration

downstream **upstream**

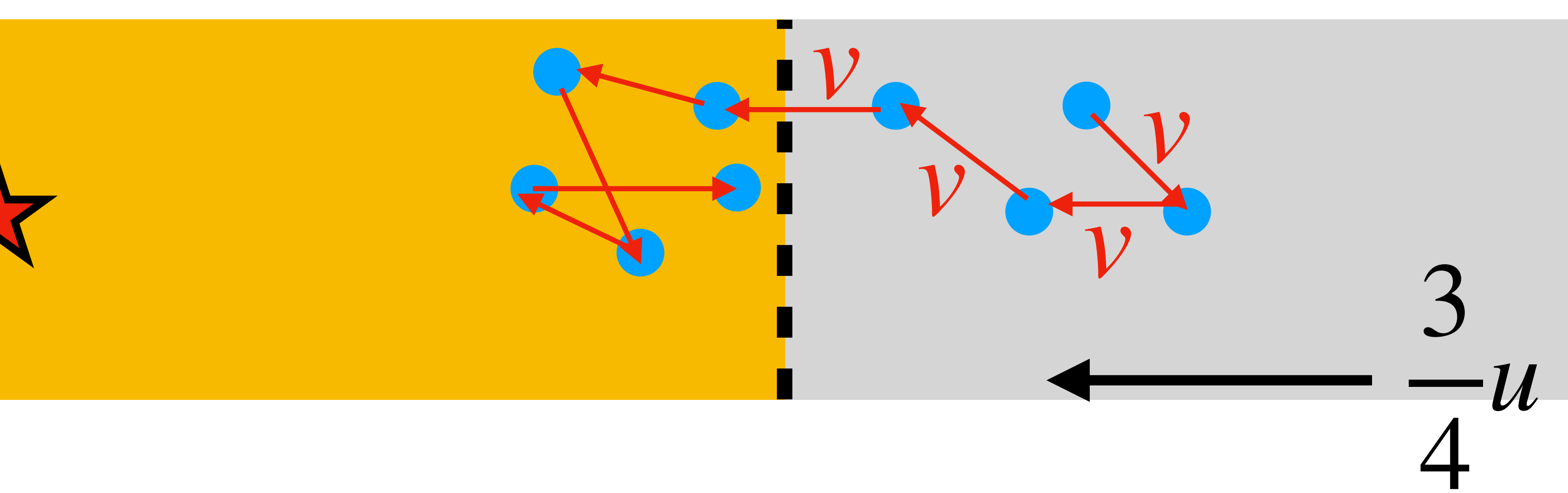
$$u << c$$

$$v \sim c$$

Upstream frame



Downstream frame



- Particles scattering on upstream side with isotropic velocity distribution in upstream rest frame.
- Eventually cross shock front.
- More scatterings on downstream side. Eventually velocity distribution becomes isotropic in *downstream* rest frame.
- Picked up bulk velocity V=(3/4)u away from explosion.

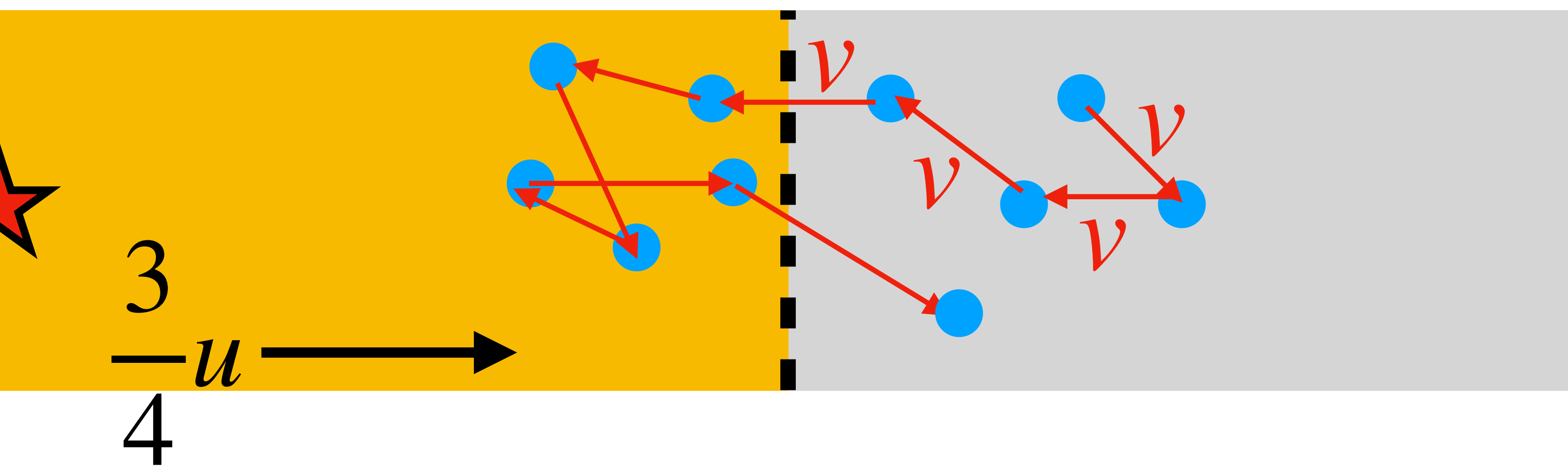
Shock acceleration

downstream **upstream**

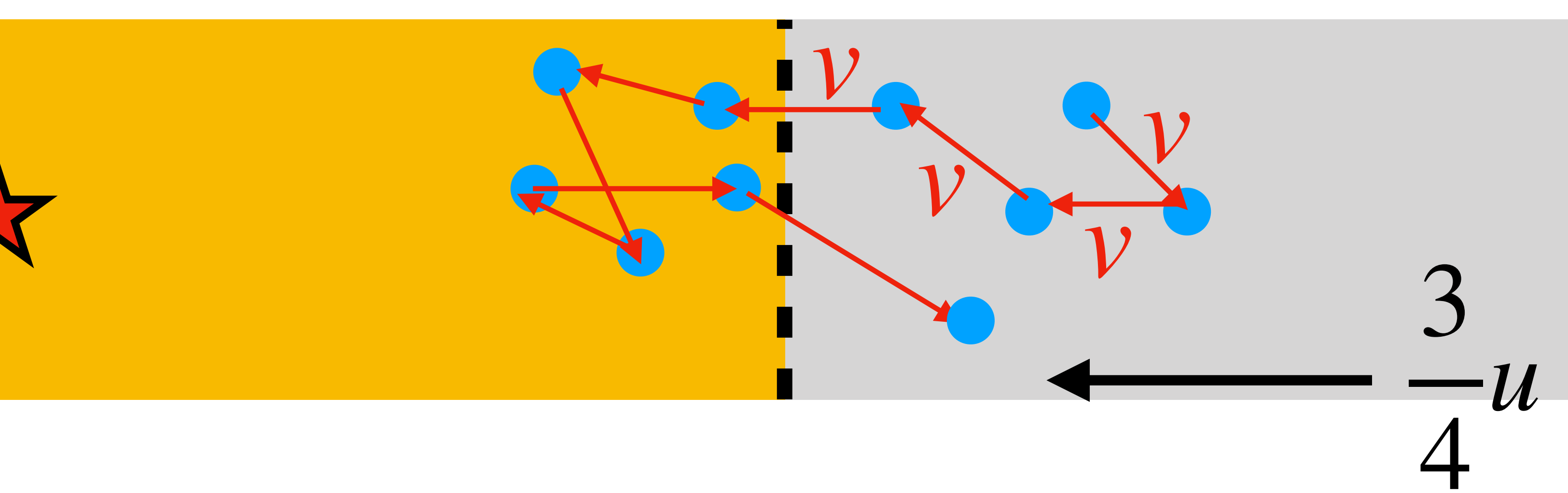
$$u << c$$

$$v \sim c$$

Upstream frame



Downstream frame



- Particles scattering on upstream side with isotropic velocity distribution in upstream rest frame.
- Eventually cross shock front.
- More scatterings on downstream side. Eventually velocity distribution becomes isotropic in *downstream* rest frame.
- Picked up bulk velocity V=(3/4)u away from explosion.
- Eventually cross shock front.

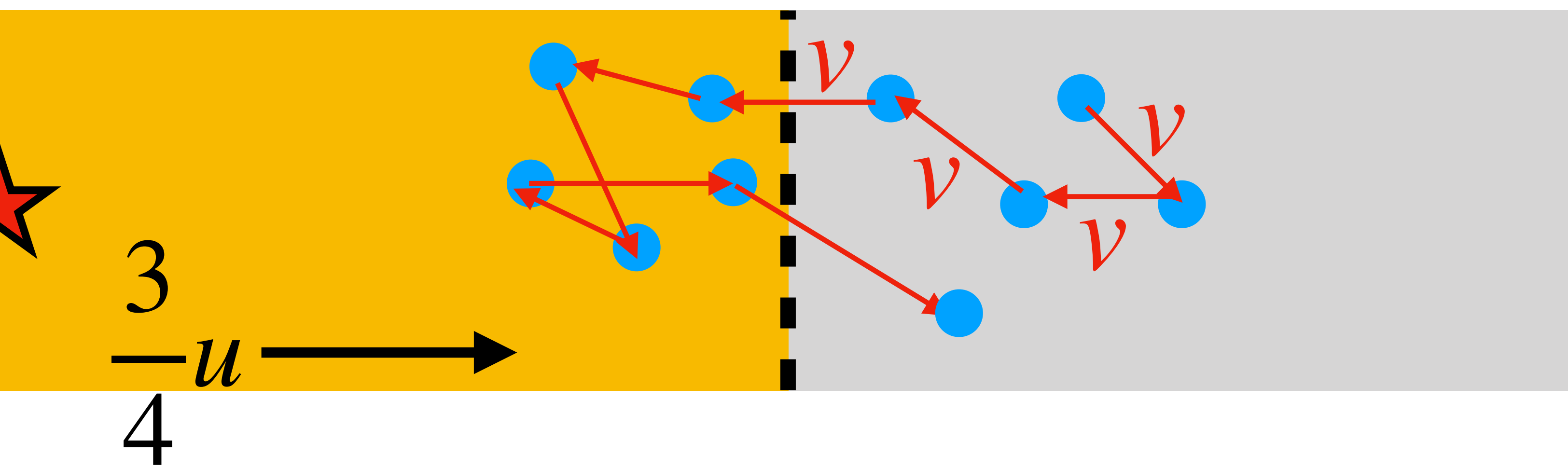
Shock acceleration

downstream **upstream**

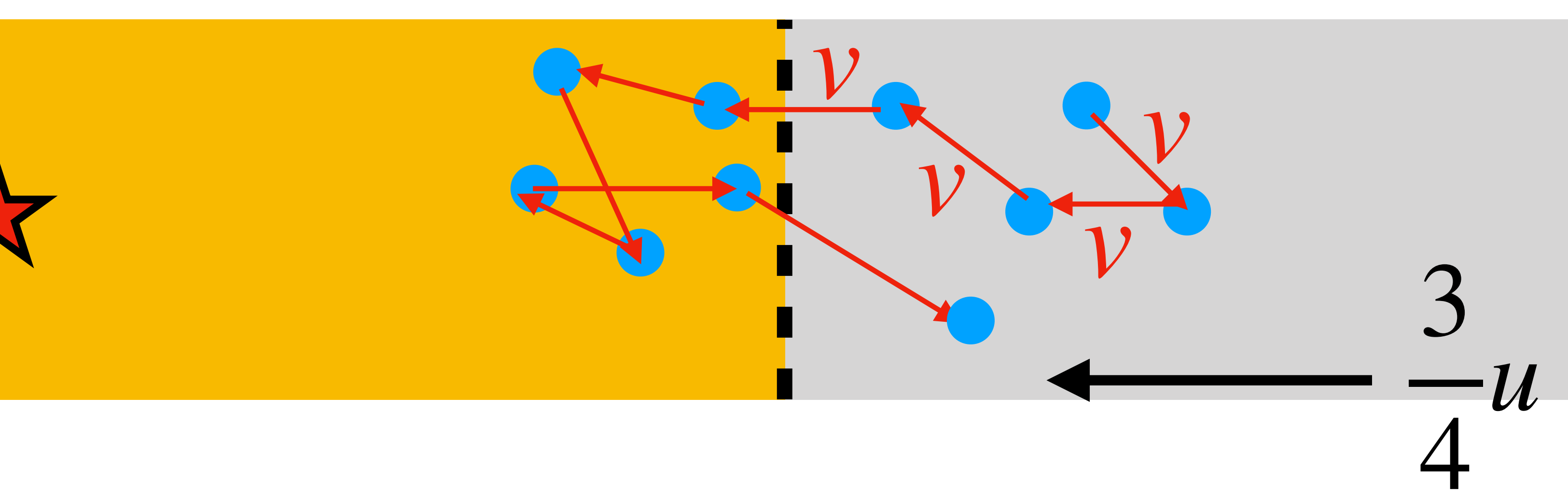
$$u << c$$

$$v \sim c$$

Upstream frame

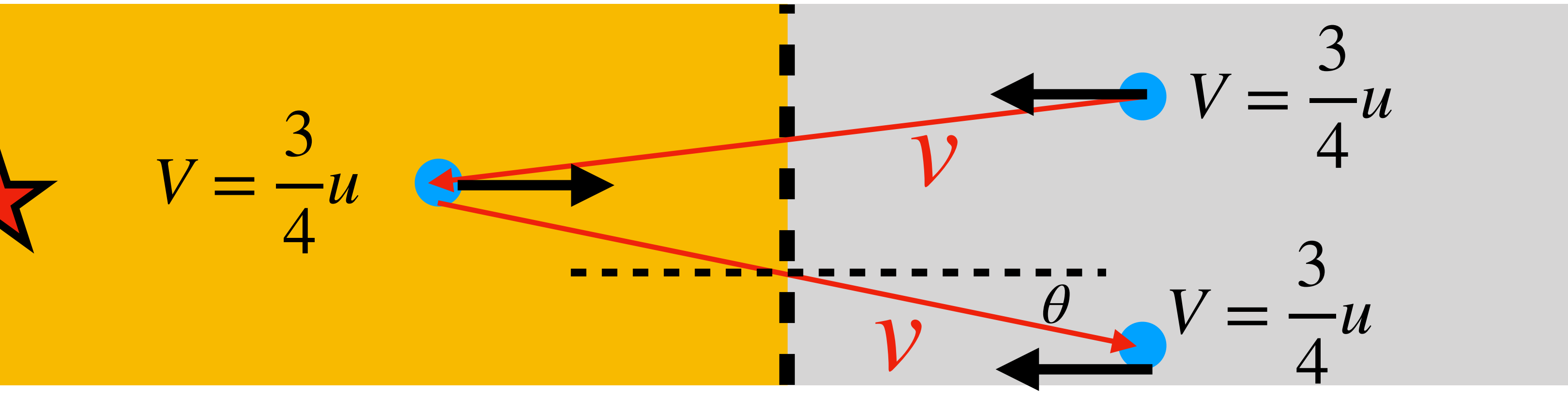


Downstream frame



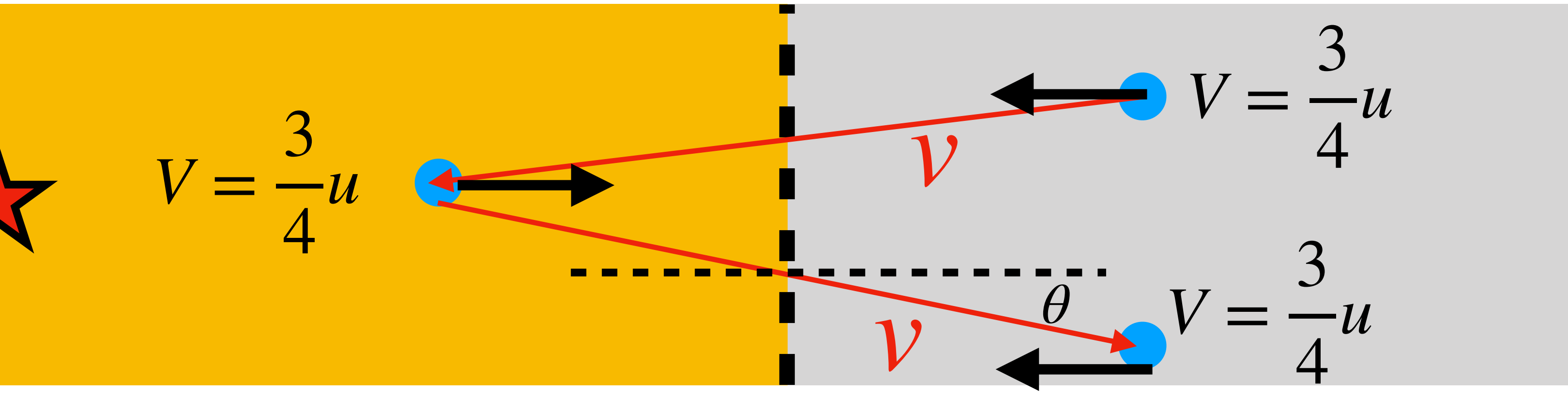
- Particles scattering on upstream side with isotropic velocity distribution in upstream rest frame.
- Eventually cross shock front.
- More scatterings on downstream side. Eventually velocity distribution becomes isotropic in *downstream* rest frame.
- Picked up bulk velocity V=(3/4)u away from explosion.
- Eventually cross shock front.
- More scatterings, pick up bulk velocity V=(3/4)u towards explosion.

Shock acceleration



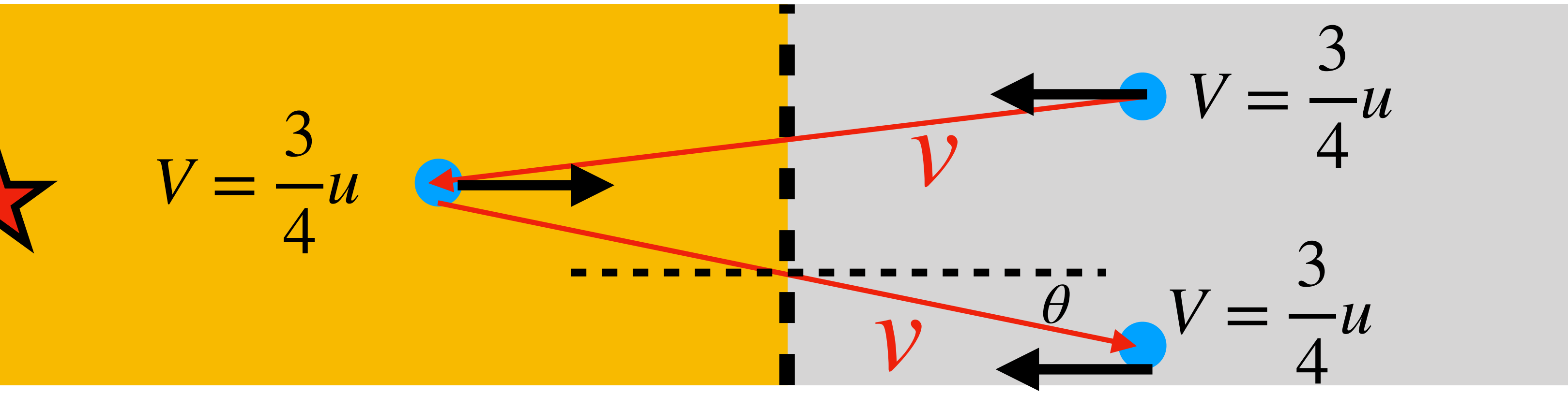
- Problem becomes like the trash compactor! Particle bouncing back-and-forth, each time picking up energy.

Shock acceleration



- Problem becomes like the trash compactor! Particle bouncing back-and-forth, each time picking up energy.
- Fractional energy increase each crossing: $\Delta E / E = (V/c)\cos\theta$

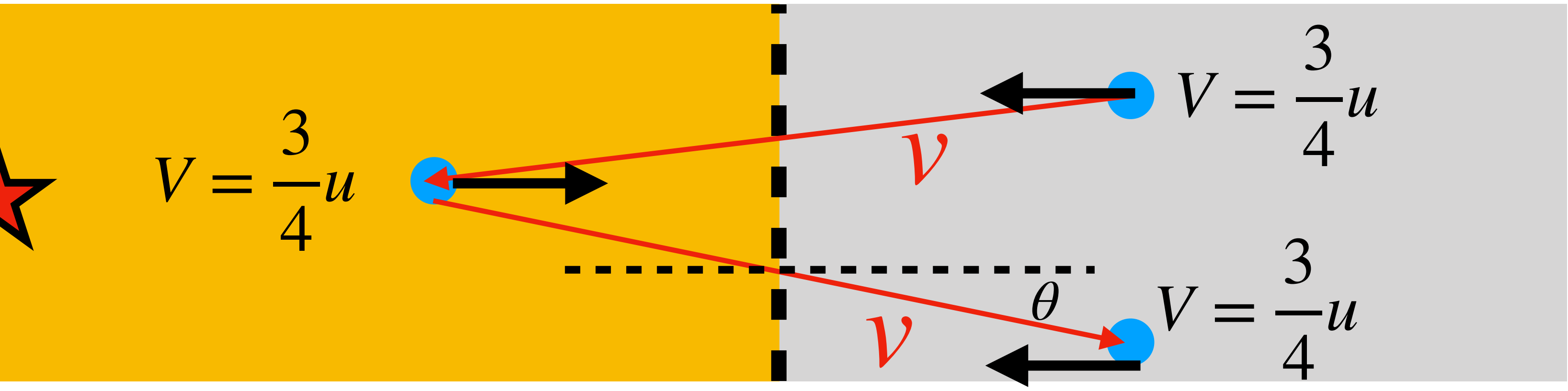
Shock acceleration



- Problem becomes like the trash compactor! Particle bouncing back-and-forth, each time picking up energy.
- Fractional energy increase each crossing: $\Delta E/E = (V/c)\cos\theta$
- Probability of crossing with angle in range θ to θ+dθ =

$$\text{solid angle subtended by } d\theta \times \text{velocity approaching shock/c}$$

Shock acceleration

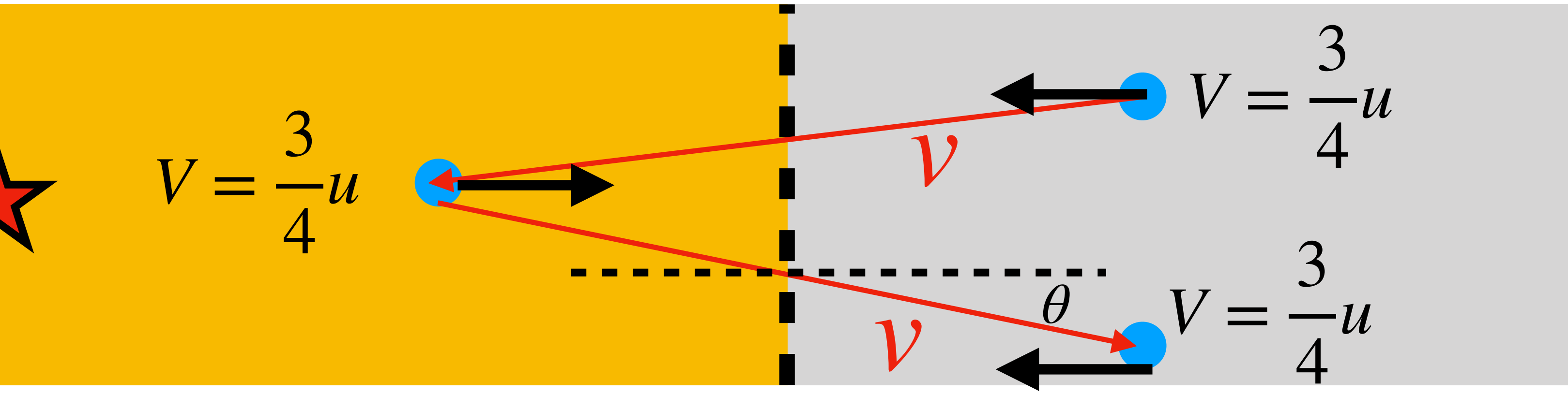


- Problem becomes like the trash compactor! Particle bouncing back-and-forth, each time picking up energy.
- Fractional energy increase each crossing: $\Delta E/E = (V/c)\cos\theta$
- Probability of crossing with angle in range θ to θ+dθ =

$$\text{solid angle subtended by d}\theta \times \text{velocity approaching shock/c}$$

$$p(\theta)d\theta \propto d\cos\theta \times (v/c)\cos\theta$$

Shock acceleration



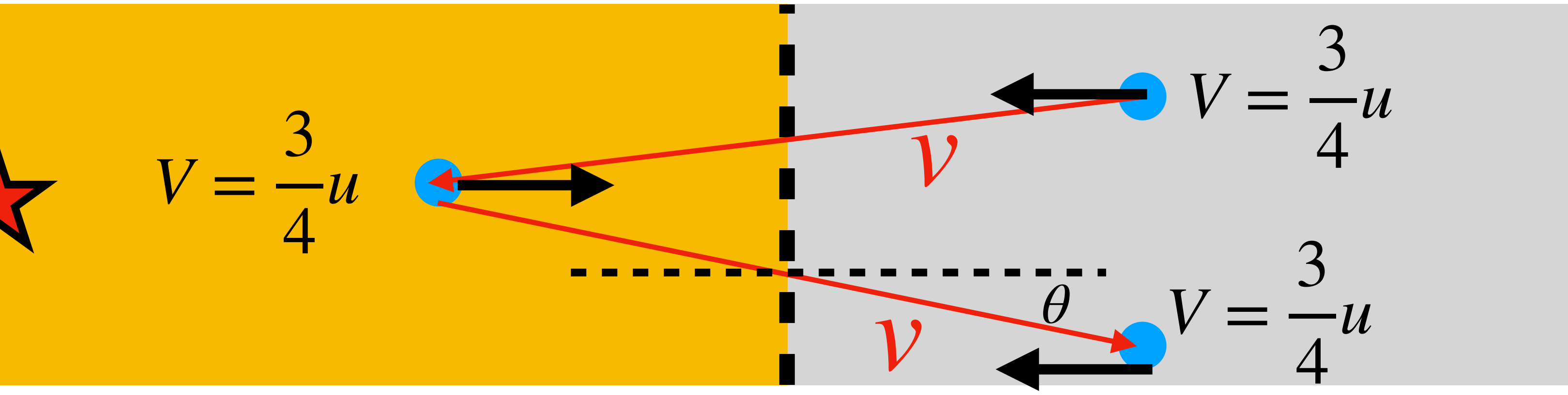
- Problem becomes like the trash compactor! Particle bouncing back-and-forth, each time picking up energy.
- Fractional energy increase each crossing: $\Delta E/E = (V/c)\cos\theta$
- Probability of crossing with angle in range θ to θ+dθ =

$$\text{solid angle subtended by d}\theta \times \text{velocity approaching shock/c}$$

$$p(\theta)d\theta \propto d\cos\theta \times (v/c)\cos\theta$$

$$\int_0^{\pi/2} p(\theta)d\theta = 1 \implies p(\theta)d\theta = 2\cos\theta d\cos\theta$$ …v~c

Shock acceleration



- Problem becomes like the trash compactor! Particle bouncing back-and-forth, each time picking up energy.
- Fractional energy increase each crossing: $\Delta E/E = (V/c)\cos\theta$
- Probability of crossing with angle in range θ to θ+dθ =

$$\text{solid angle subtended by } d\theta \times \text{velocity approaching shock/c}$$

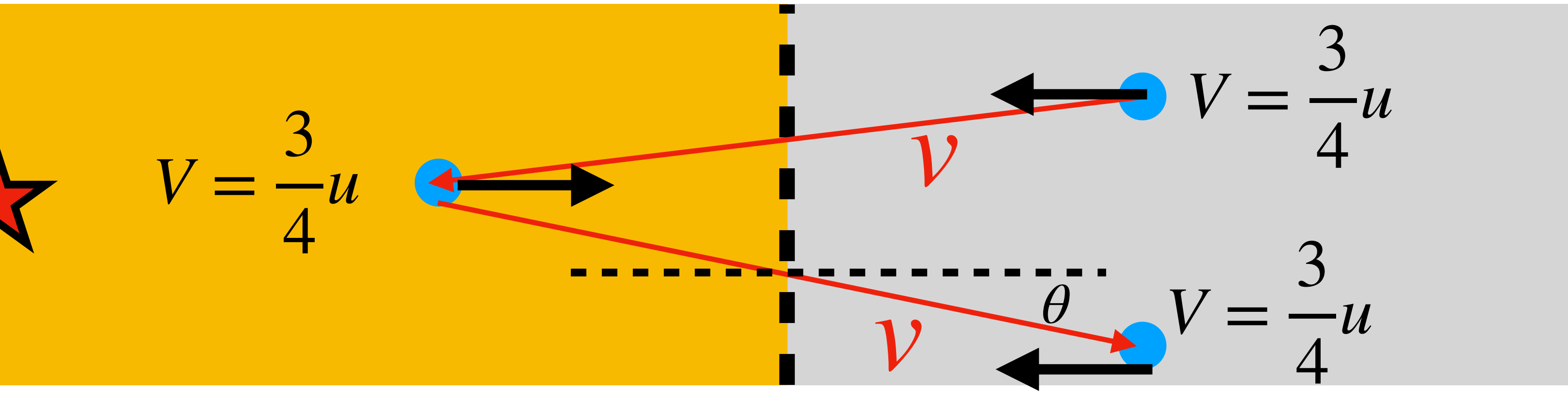
$$p(\theta)d\theta \propto d\cos\theta \times (v/c)\cos\theta$$

$$\int_0^{\pi/2} p(\theta)d\theta = 1 \implies p(\theta)d\theta = 2\cos\theta \; d\cos\theta$$ ...v~c

- Average fractional energy gain per crossing:

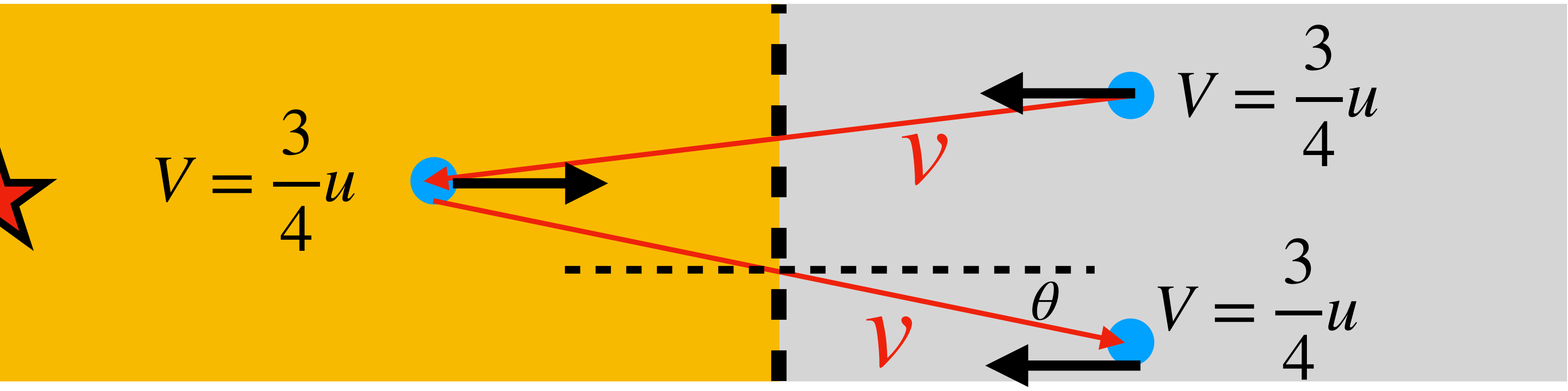
$$\langle \Delta E/E \rangle = \int_0^{\pi/2} \frac{V}{c}\cos\theta p(\theta) \; d\theta = 2\frac{V}{c}\int_0^1 \cos^2\theta \; d\cos\theta = \frac{2}{3}\frac{V}{c}$$

Shock acceleration



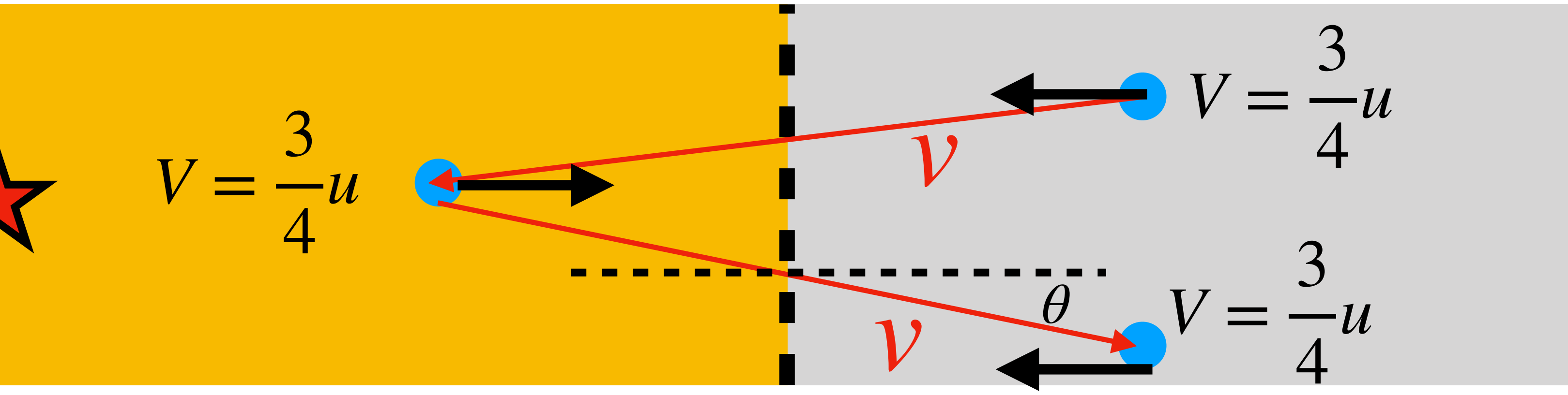
- Average fractional energy gain per crossing:

Shock acceleration



- Average fractional energy gain per crossing: $\langle \Delta E/E \rangle = \frac{2}{3}\frac{V}{c}$
- Average fractional energy gain per round trip: $\langle \Delta E/E \rangle = \frac{4}{3}\frac{V}{c} = \frac{u}{c}$
- Therefore: $\beta = 1 + \frac{u}{c}$

Shock acceleration

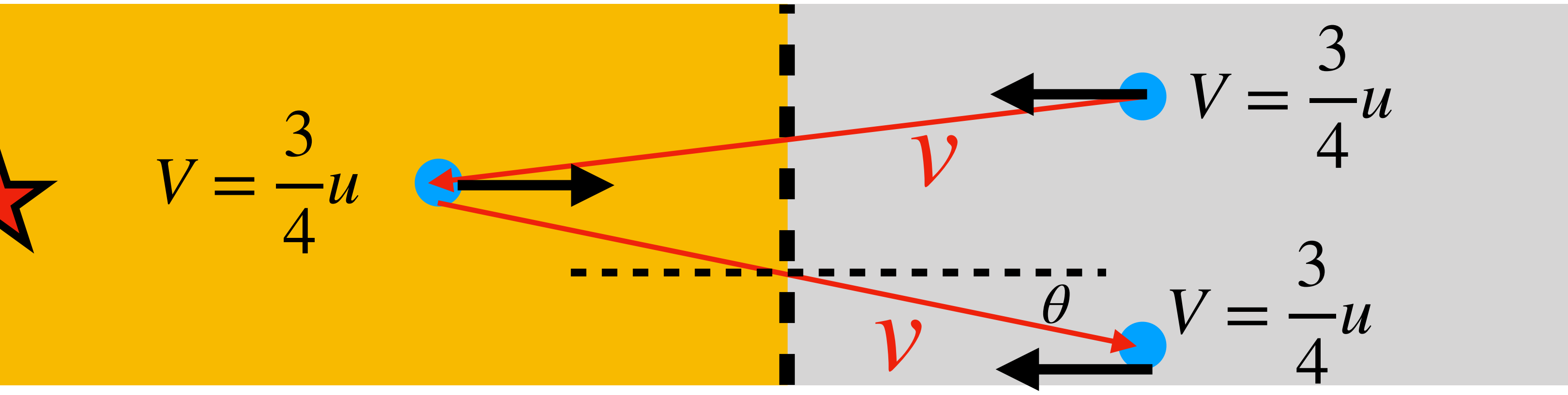


- Average fractional energy gain per crossing: $\langle \Delta E/E \rangle = \frac{2}{3}\frac{V}{c}$

- Average fractional energy gain per round trip: $\langle \Delta E/E \rangle = \frac{4}{3}\frac{V}{c} = \frac{u}{c}$
- Therefore: $\beta = 1 + \frac{u}{c}$

Because fractional energy gain goes as (u/c) instead of $(u/c)^2$, this diffusive shock acceleration process is known as *first order Fermi acceleration*. Now much easier to accelerate particles to the observed $E \sim 10^{20}$ eV!

Shock acceleration



- Average fractional energy gain per crossing: $\langle \Delta E/E \rangle = \frac{2}{3}\frac{V}{c}$
- Average fractional energy gain per round trip: $\langle \Delta E/E \rangle = \frac{4}{3}\frac{V}{c} = \frac{u}{c}$
- Therefore: $\beta = 1 + \frac{u}{c}$

Because fractional energy gain goes as (u/c) instead of $(u/c)^2$, this diffusive shock acceleration process is known as *first order Fermi acceleration*. Now much easier to accelerate particles to the observed E~10^{20} eV!

Fermi did not come up with the shock front idea though, this dates back to the late 1970s, with many of the key insights being made by Tony Bell (Oxford): https://ui.adsabs.harvard.edu/abs/1978MNRAS.182..147B/abstract; https://ui.adsabs.harvard.edu/abs/1978MNRAS.182..443B/abstract

Shock acceleration

$$\frac{dN}{dE} \propto E^{\ln(P)/\ln(\beta)-1}$$

$$\beta = 1 + \frac{u}{c}$$

To get power-law index, must calculate probability of particles remaining near the shock region after each round trip, P.

Shock acceleration

$$\frac{dN}{dE} \propto E^{\ln(P)/\ln(\beta)-1}$$

$$\beta = 1 + \frac{u}{c}$$

To get power-law index, must calculate probability of particles remaining near the shock region after each round trip, P.

- Each crossing, a fraction u/c of the particles are swept away (i.e. advected) from the shock front by the bulk motion of downstream particles away from shock front (or the motion of the shock front away from the downstream particles — depending on what reference frame we are considering).
- Therefore the probability of a particle sticking around for another pair of crossings after each round trip is P=1-u/c.

Shock acceleration

$$\frac{dN}{dE} \propto E^{\ln(P)/\ln(\beta)-1}$$

$$\beta = 1 + \frac{u}{c}$$

To get power-law index, must calculate probability of particles remaining near the shock region after each round trip, P.

- Each crossing, a fraction u/c of the particles are swept away (i.e. advected) from the shock front by the bulk motion of downstream particles away from shock front (or the motion of the shock front away from the downstream particles — depending on what reference frame we are considering).
- Therefore the probability of a particle sticking around for another pair of crossings after each round trip is P=1-u/c.

$$\frac{\ln(P)}{\ln(\beta)} = \frac{\ln(1-u/c)}{\ln(1+u/c)} \approx \frac{-u/c}{u/c} = -1$$

Taylor expansion around u=0

Shock acceleration

$$\frac{dN}{dE} \propto E^{\ln(P)/\ln(\beta)-1}$$

$$\beta = 1 + \frac{u}{c}$$

To get power-law index, must calculate probability of particles remaining near the shock region after each round trip, P.

- Each crossing, a fraction u/c of the particles are swept away (i.e. advected) from the shock front by the bulk motion of downstream particles away from shock front (or the motion of the shock front away from the downstream particles — depending on what reference frame we are considering).
- Therefore the probability of a particle sticking around for another pair of crossings after each round trip is P=1-u/c.

Taylor expansion around u=0

$$\frac{\ln(P)}{\ln(\beta)} = \frac{\ln(1-u/c)}{\ln(1+u/c)} \approx \frac{-u/c}{u/c} = -1$$

$$\therefore \frac{dN}{dE} \propto E^{-2}$$

Shock acceleration

$$\therefore \frac{dN}{dE} \propto E^{-2}$$

- Close to observed $E^{-2.5}$, but not exactly!
- Still active area of research.
- Mechanism so popular because it explains why you can get the *same* power-law spectrum for particles accelerated in diverse array of astrophysical objects (e.g. supernova remnants, AGN etc). There only needs to be a strong shock!