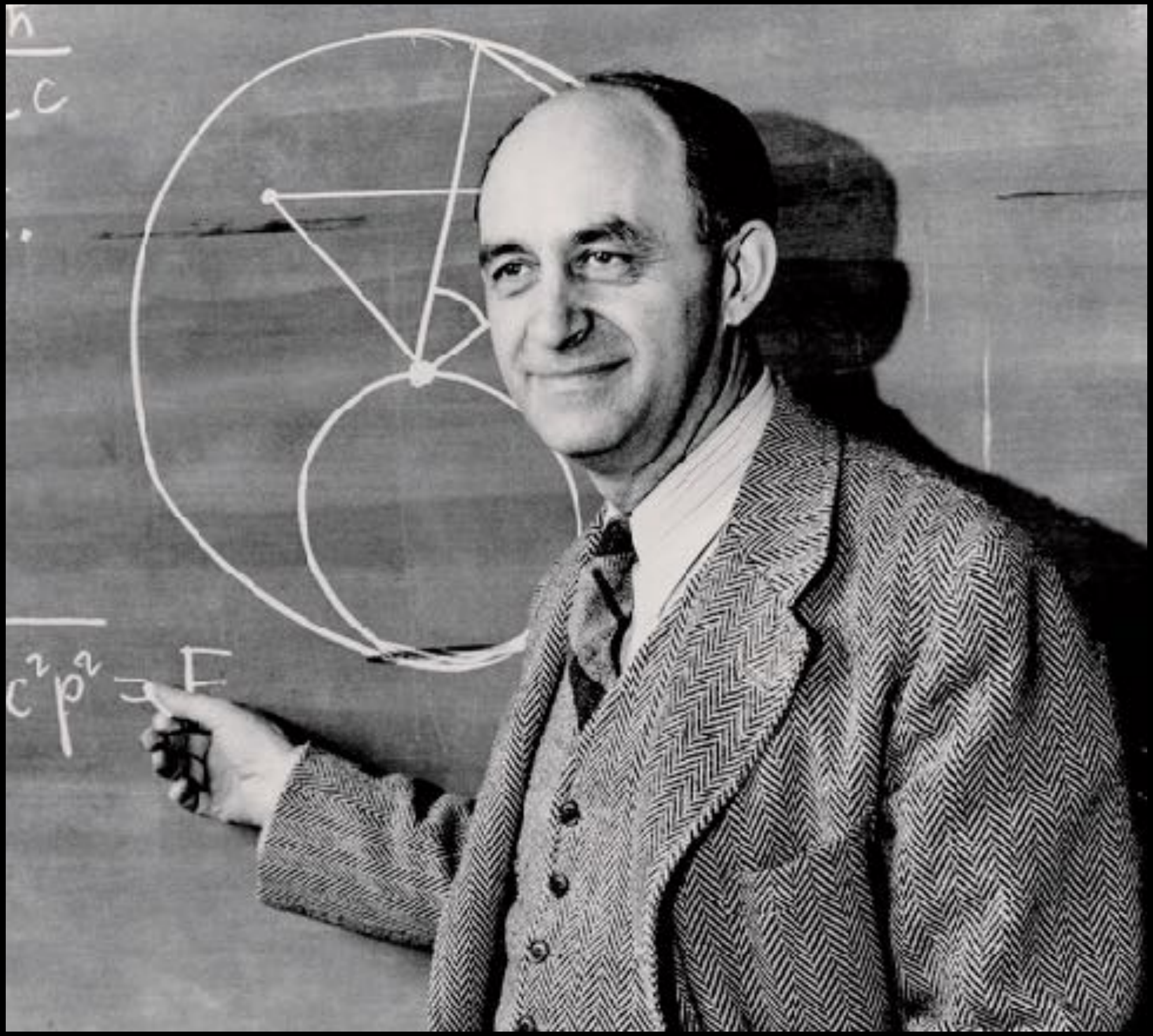
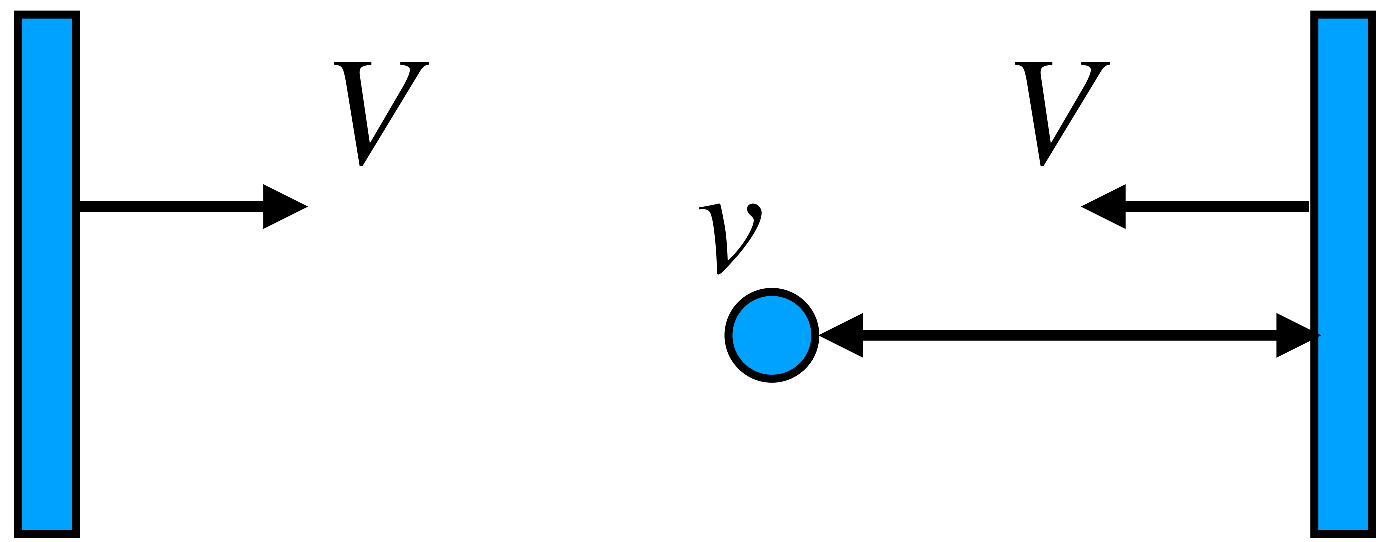
Lecture 2
Shock Acceleration - Correction



Enrico Fermi

Fermi acceleration



Assume: $V \ll c$ $\qquad v \sim c$ $\qquad$ No recoil of mirrors

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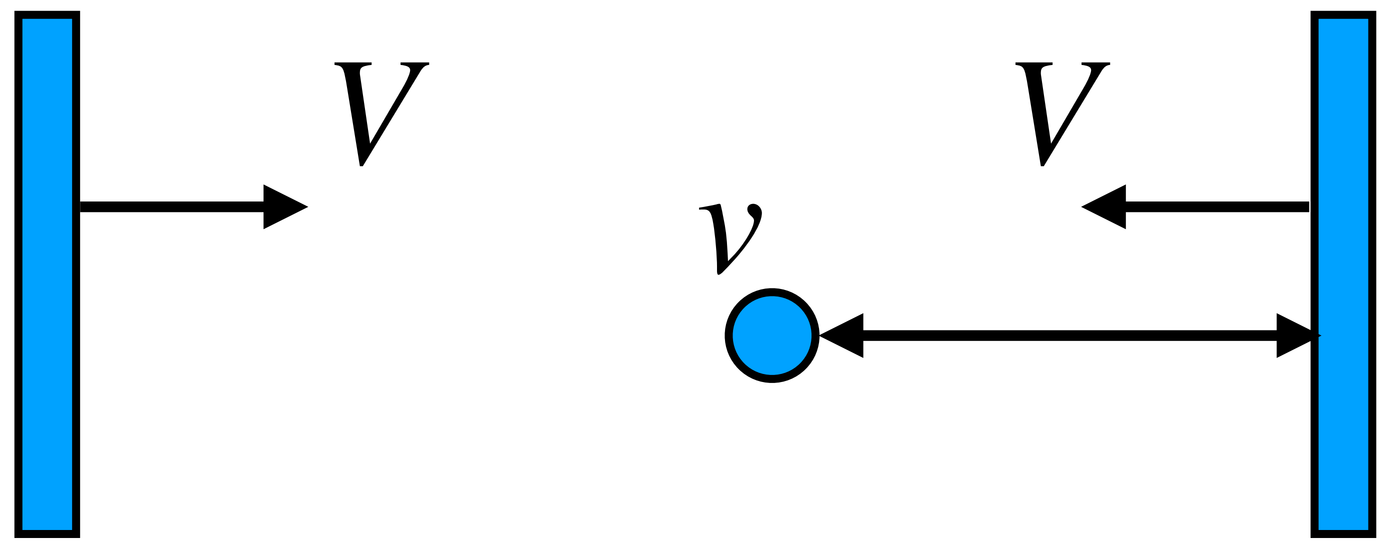
Momentum increase per collision $= \gamma m V$ $\qquad \gamma = \dfrac{1}{\sqrt{1-(V/c)^2}}$

$$E^2 = (pc)^2 + (m_o c^2)^2$$

$$\text{Energy increase per collision} \approx \gamma m V c = \gamma m c^2 \frac{V}{c} = \frac{V}{c} E$$

$$\therefore \; \beta \equiv \frac{\text{Energy after collision}}{\text{Energy before collision}} = 1 + \frac{\Delta E}{E} \approx 1 + \frac{V}{c}$$

Fermi acceleration



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Nonsense!

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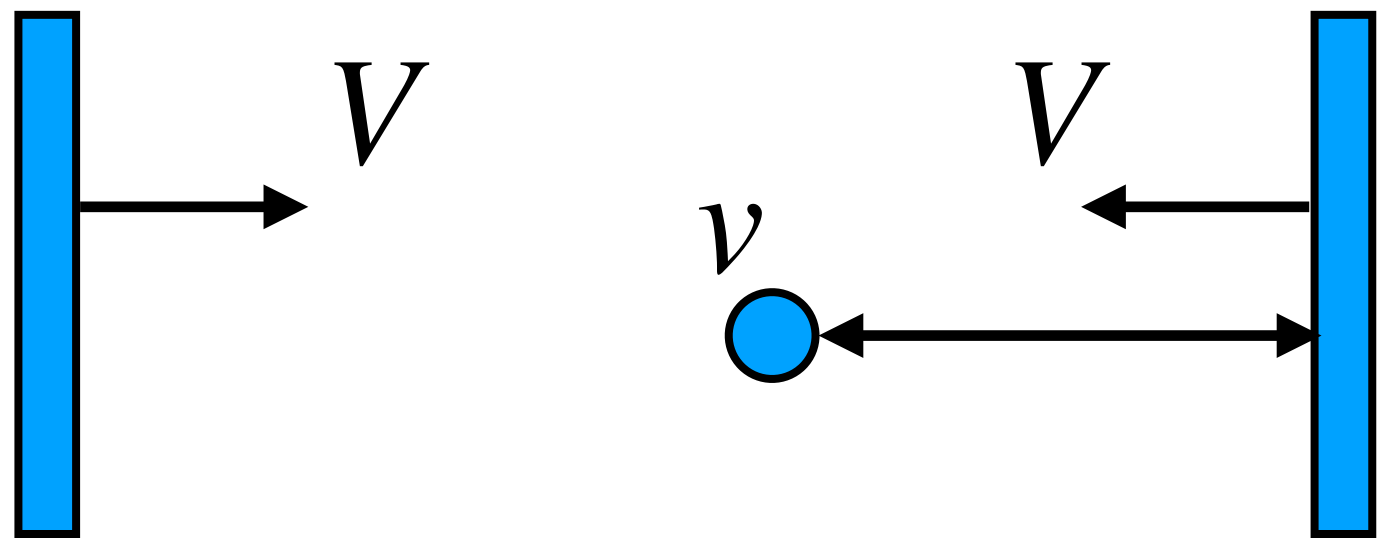


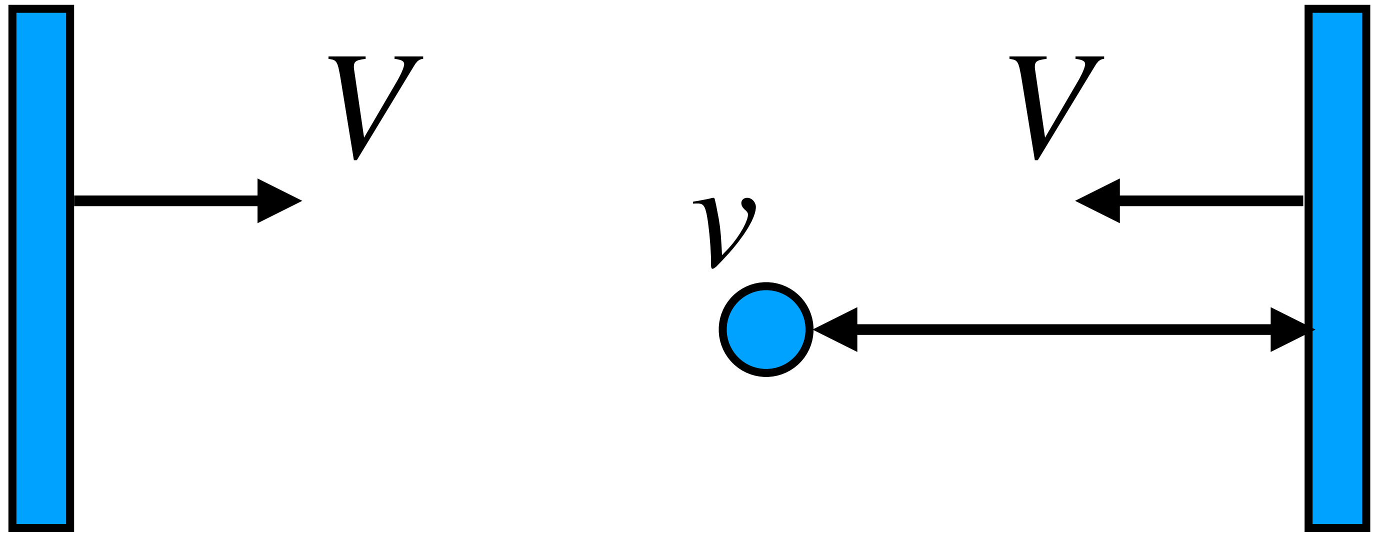
$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Assume: $V \ll c$ $v \sim c$ No recoil of mirrors

$$p_{\text{before}} = \frac{mv}{\sqrt{1-(v/c)^2}}$$

$$p_{\text{after}} = \frac{m(v+V)}{\sqrt{1-([v+V]/c)^2}} \approx \frac{m(v+V)}{\sqrt{1-(v/c)^2}}$$

Fermi acceleration



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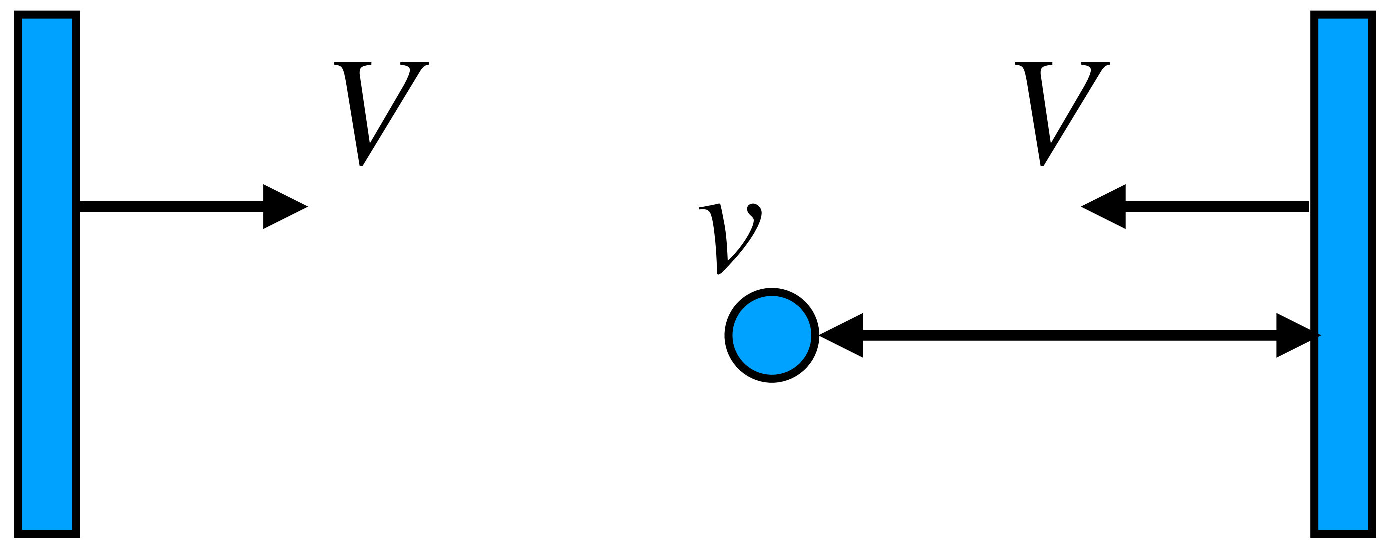

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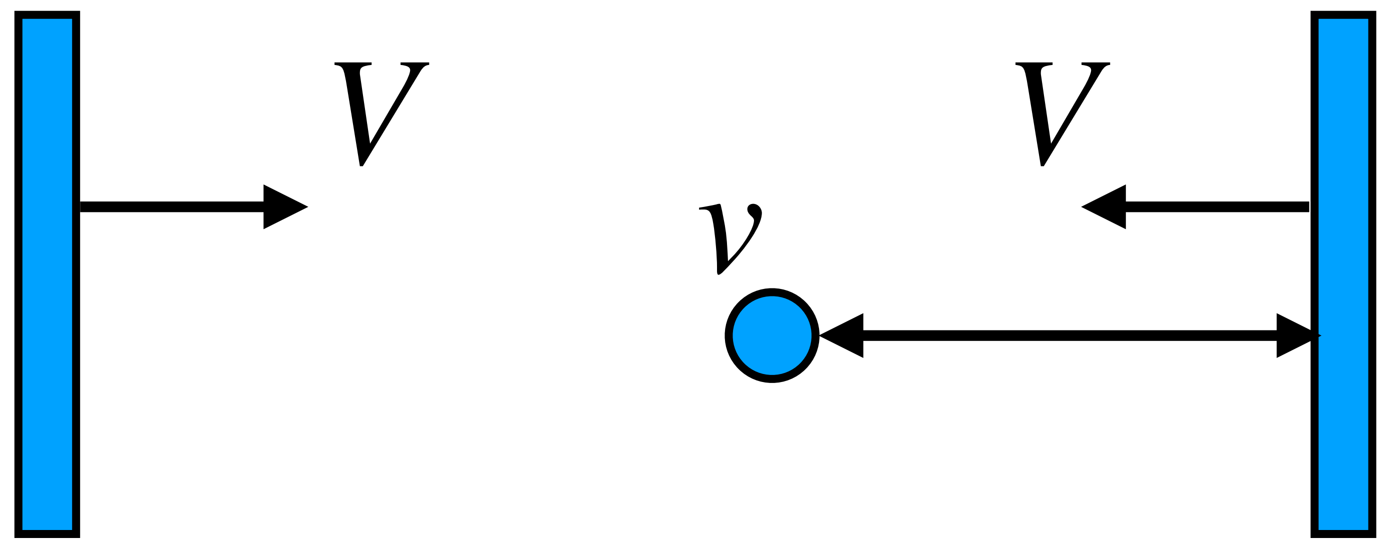
$$\therefore \;\; \Delta E = \sqrt{(\Delta p \; c)^2 + (mc^2)^2} \approx \Delta p \; c \approx \gamma m V c = \gamma m c^2 V/c \approx E \; V/c$$

Fermi acceleration



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Shock acceleration

$$\frac{dN}{dE} \propto E^{\ln(P)/\ln(\beta)-1}$$

$$\beta = 1 + \frac{u}{c}$$

To get power-law index, must calculate probability of particles remaining near the shock region after each round trip, P.

- Each crossing, a fraction u/c of the particles are swept away (i.e. advected) from the shock front by the bulk motion of downstream particles away from shock front (or the motion of the shock front away from the downstream particles — depending on what reference frame we are considering).
- Therefore the probability of a particle sticking around for another pair of crossings after each round trip is P=1-u/c.

Taylor expansion around u=0

$$\frac{\ln(P)}{\ln(\beta)} = \frac{\ln(1-u/c)}{\ln(1+u/c)} \approx \frac{-u/c}{u/c} = -1$$

$$\therefore \frac{dN}{dE} \propto E^{-2}$$

Advection

Why is a fraction u/c of particles swept away downstream each round trip?

- N = number density of particles. Rate of particles crossing and re-crossing the shock per unit area per unit time: $= \frac{1}{4} N v$

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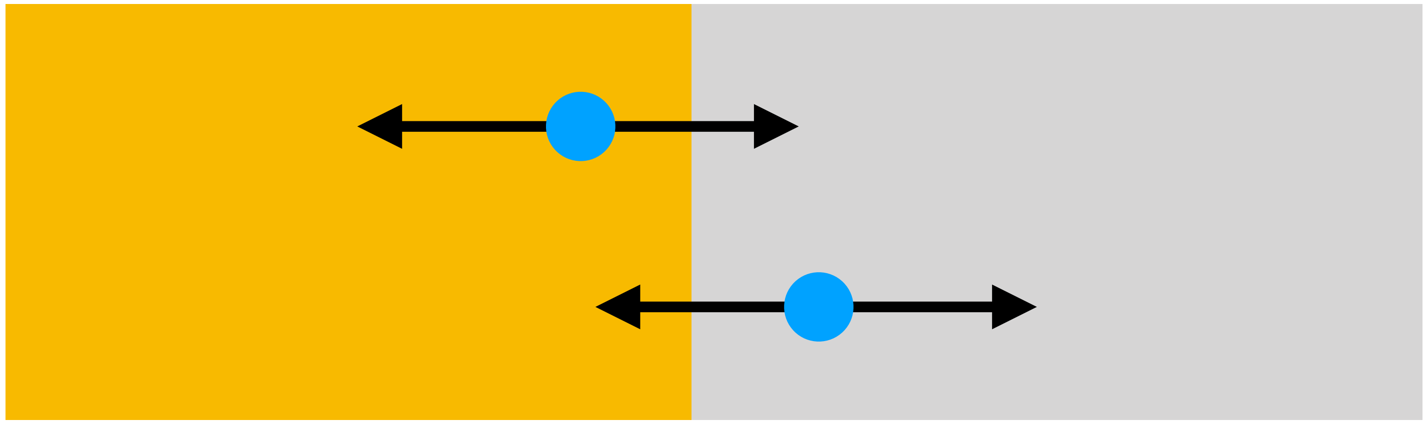

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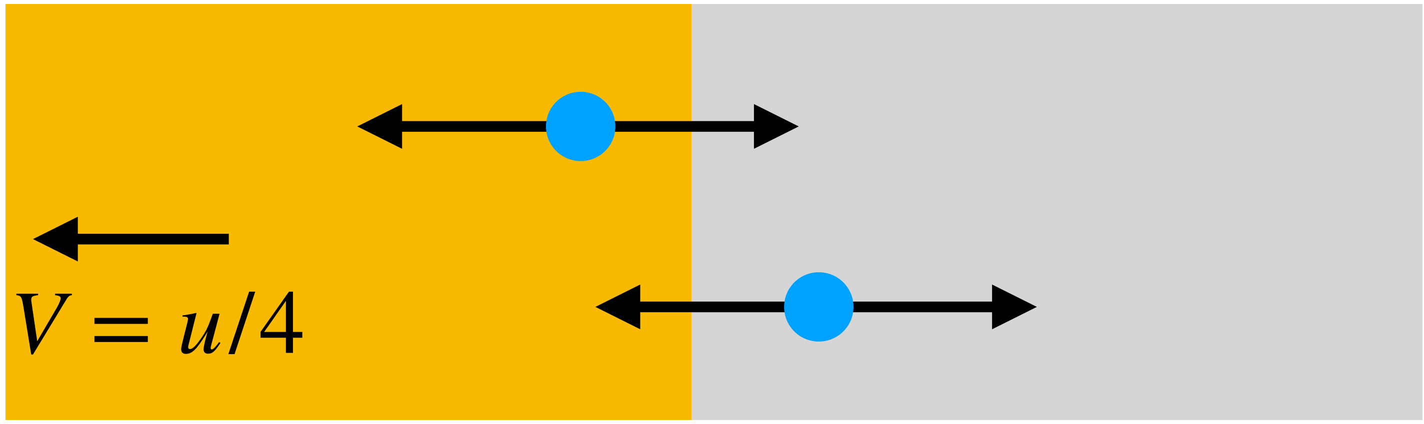

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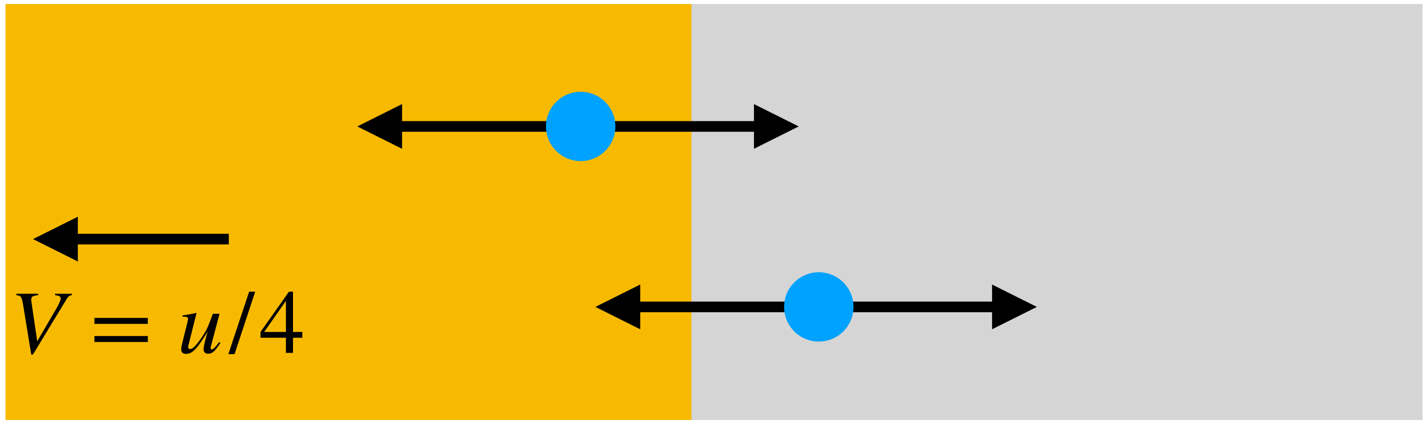

- Rate of particles advected downstream per unit area: $= NV = \frac{1}{4} N u$
- Therefore, fraction of particles crossing the shock that are advected:

$$= \frac{N\ u/4}{N\ v/4} = \frac{u}{v} \approx \frac{u}{c}$$