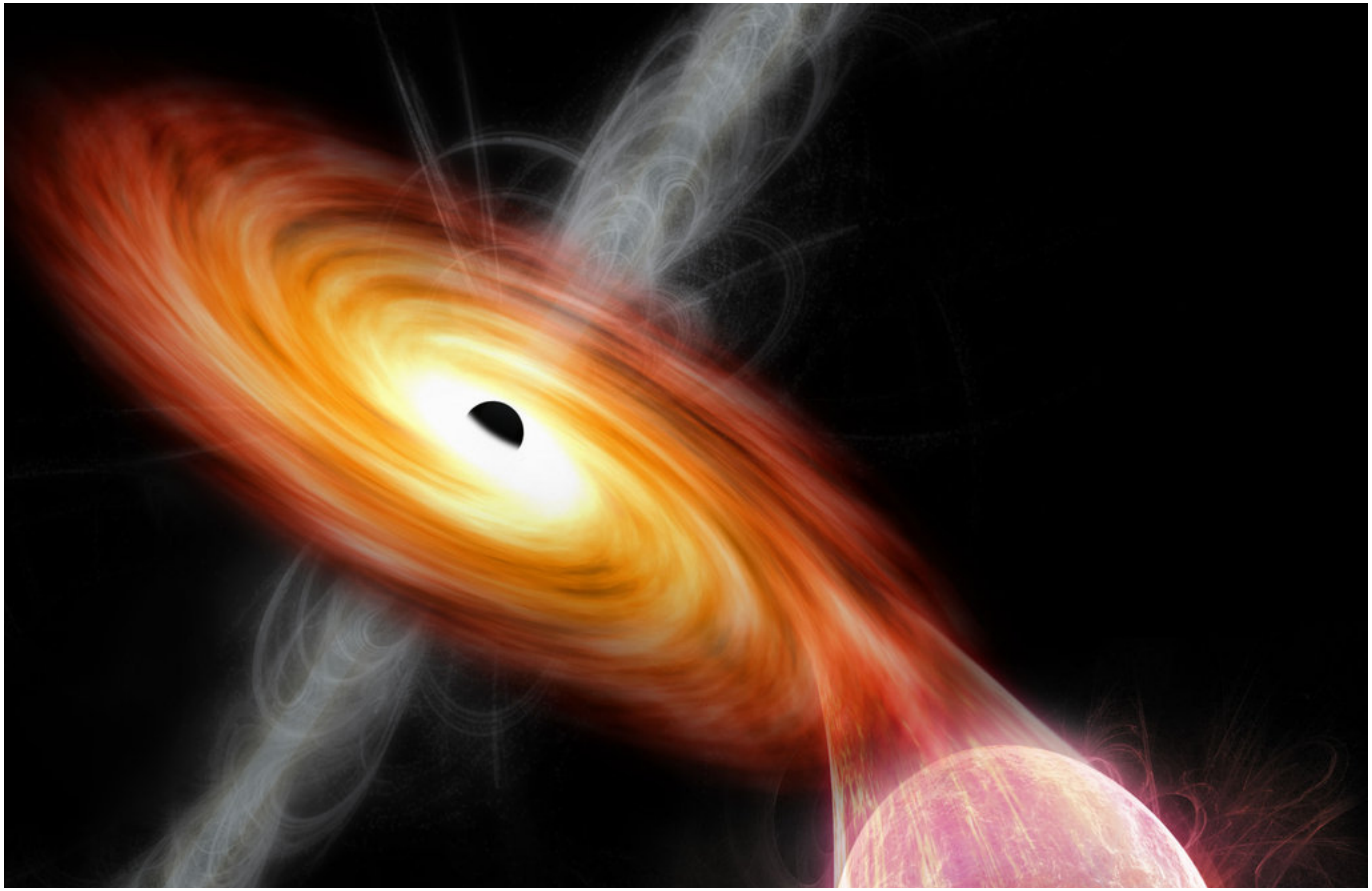
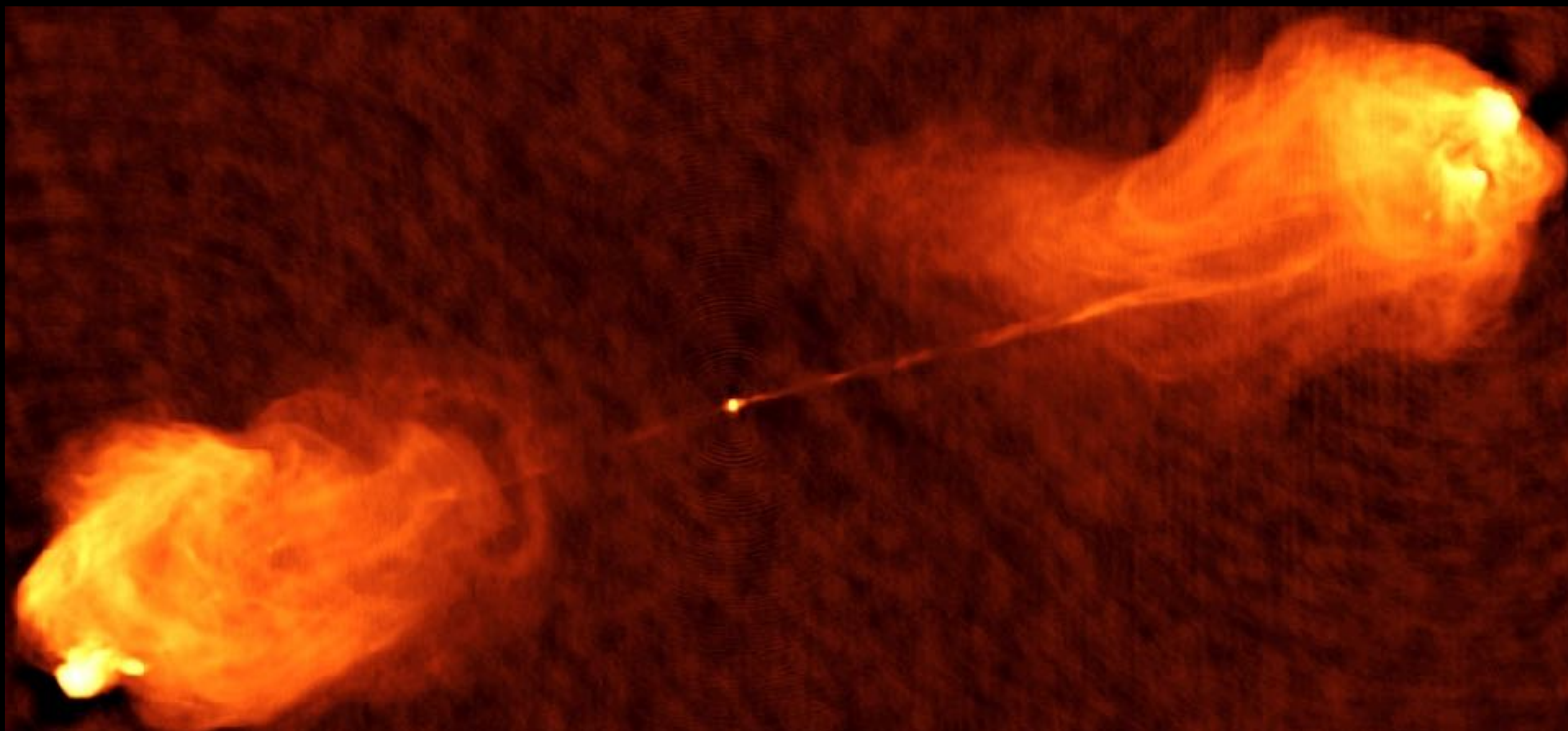
High Energy Astrophysics

Dr. Adam Ingram

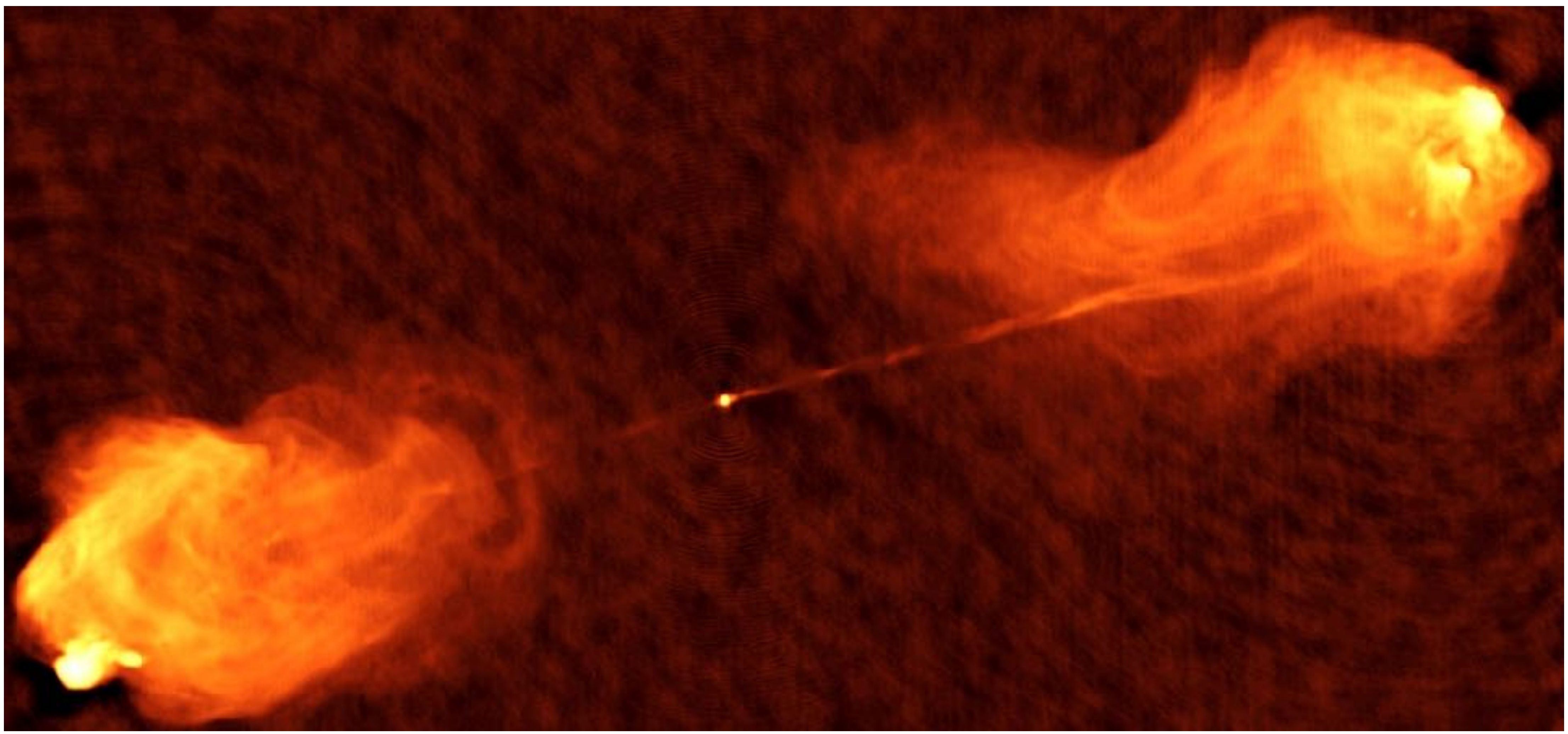


Lecture 3
Synchrotron Radiation



Introduction

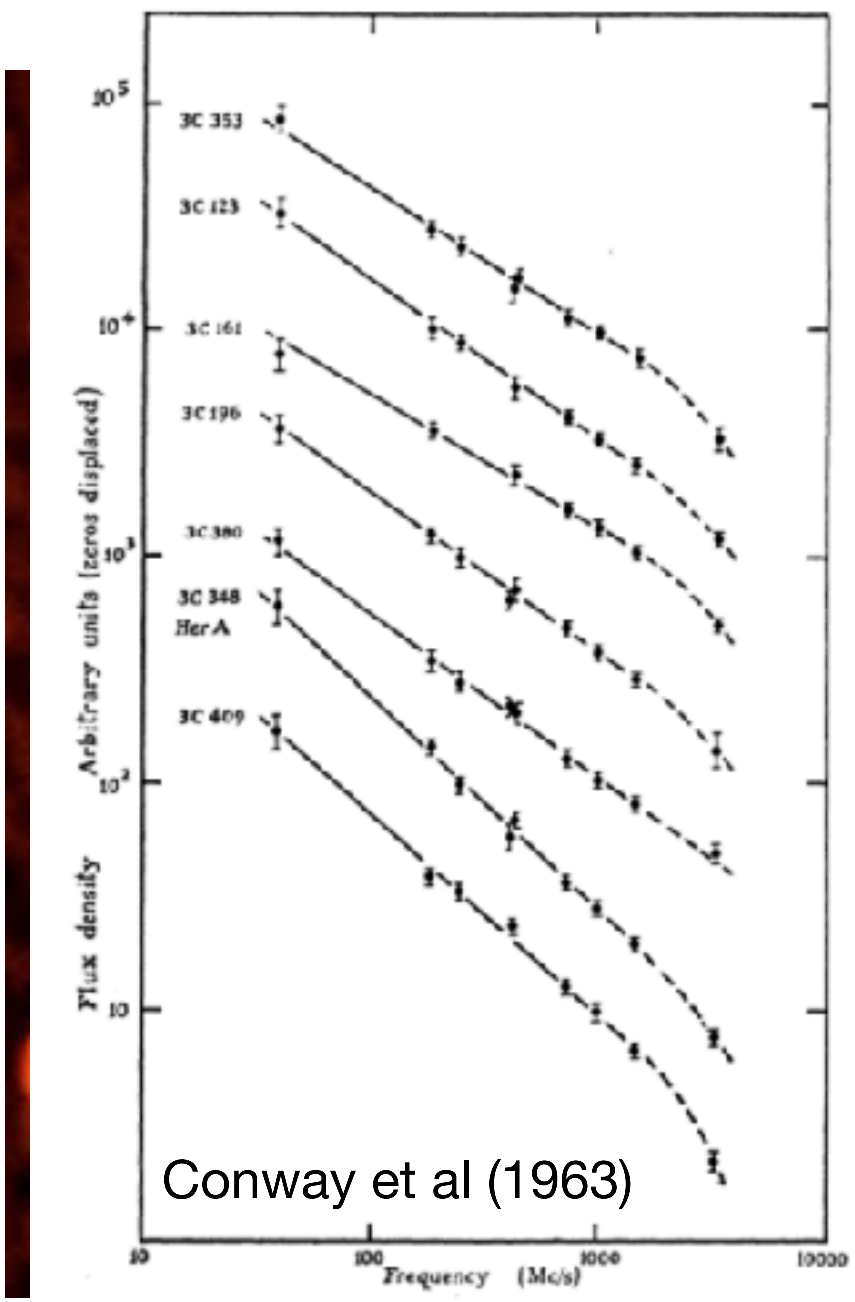
- Particles accelerated to ultra-relativistic energies at shock fronts (e.g. jet lobes, supernova remnants)
- Accelerated electrons will spiral around B-field -> *synchrotron radiation*.
- Observe strong radio emission from such regions.
- e.g. AGN/XRB jet lobes, SN remnants, galaxies (sum of radio emission from SN remnants & XRBs)



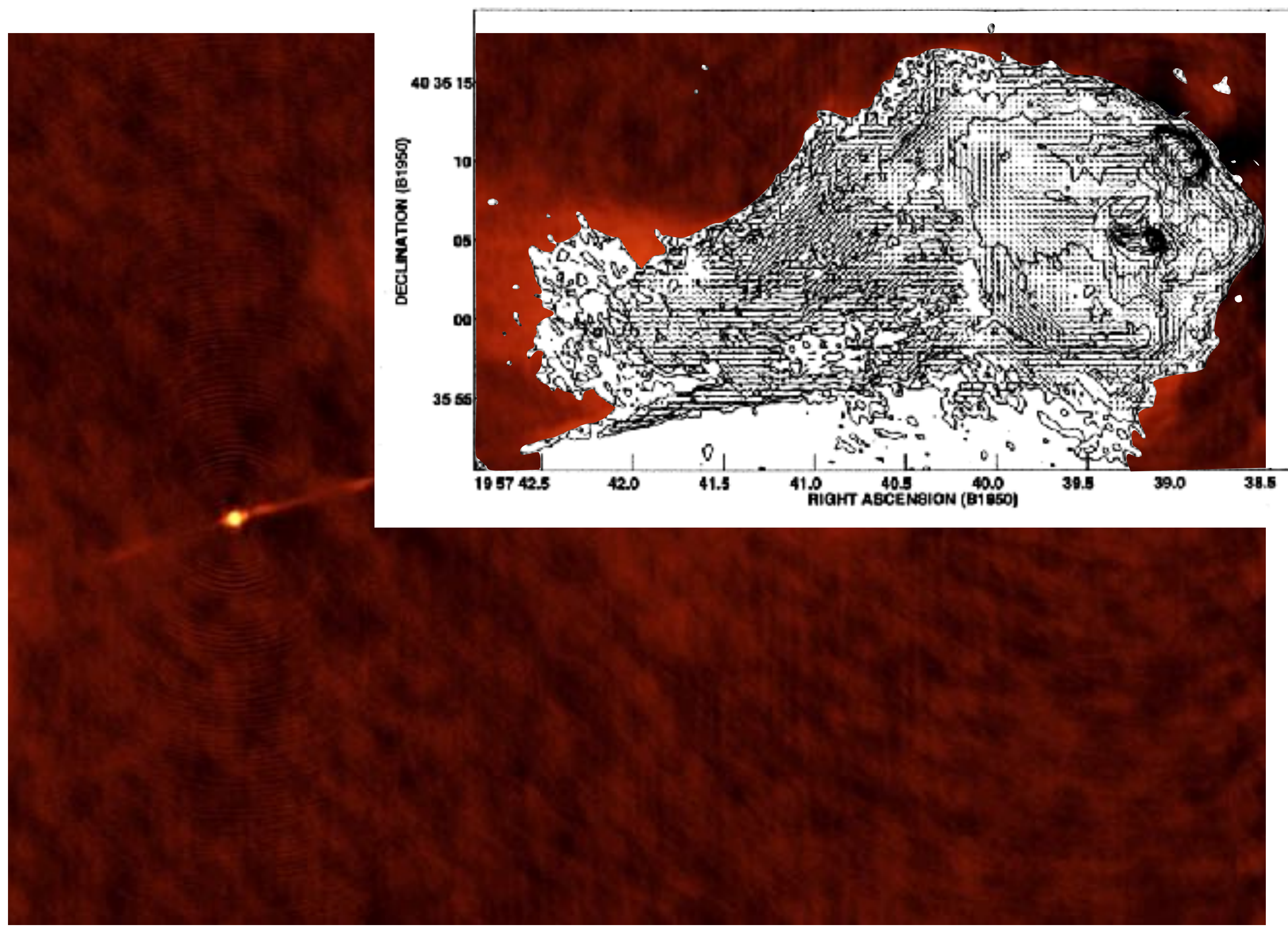
Radio galaxy Cygnus A at 5 GHz (VLA: Carilli and Barthel 1996, A&A Reviews)

Evidence for synchrotron

- Smooth, featureless broadband spectrum over many orders of magnitude in frequency;
- Power-law spectrum (will address turn-over next time);
- High degree of linear polarisation.

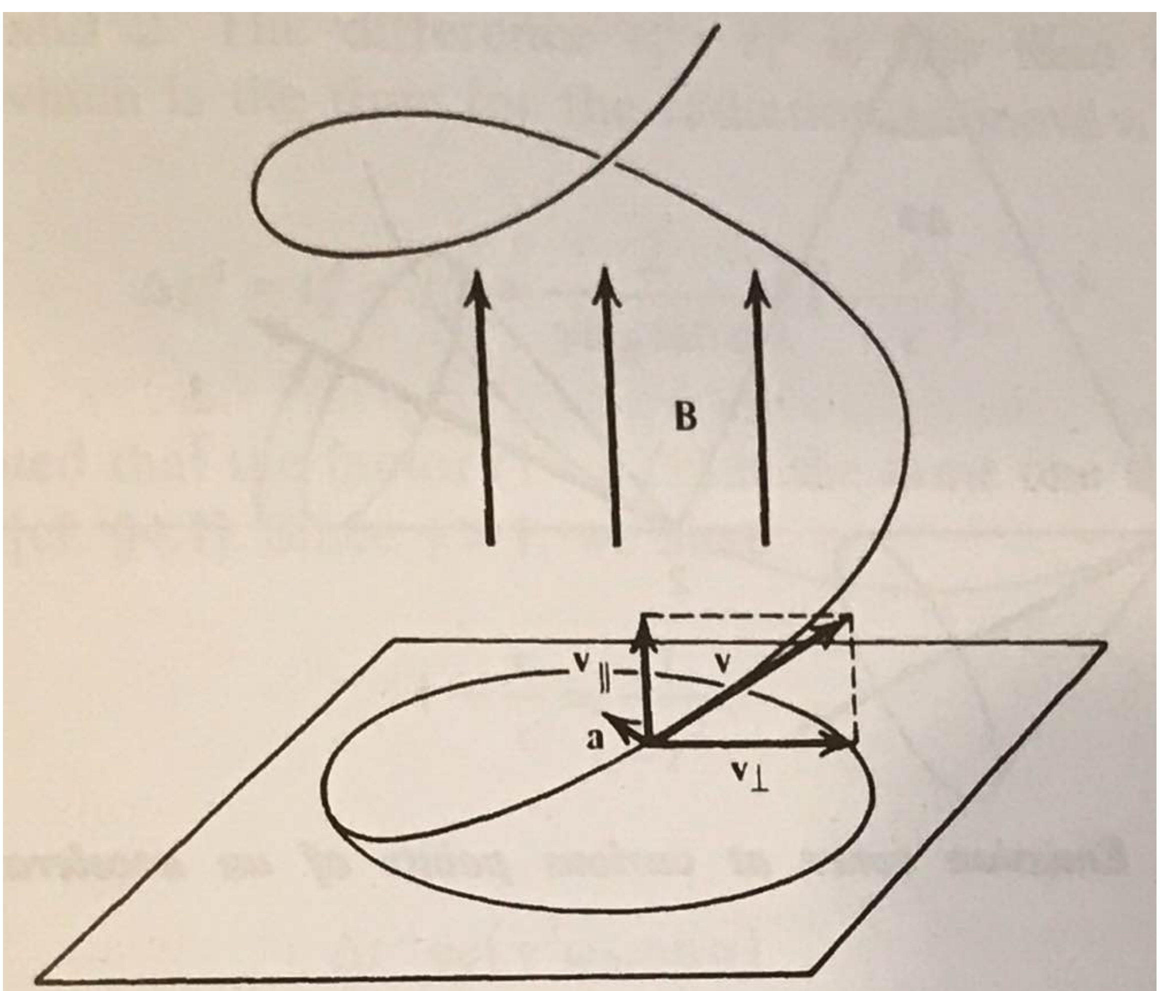


Conway et al (1963)



Synchrotron radiation

- Radiation released by relativistic electrons spiralling around magnetic field lines.



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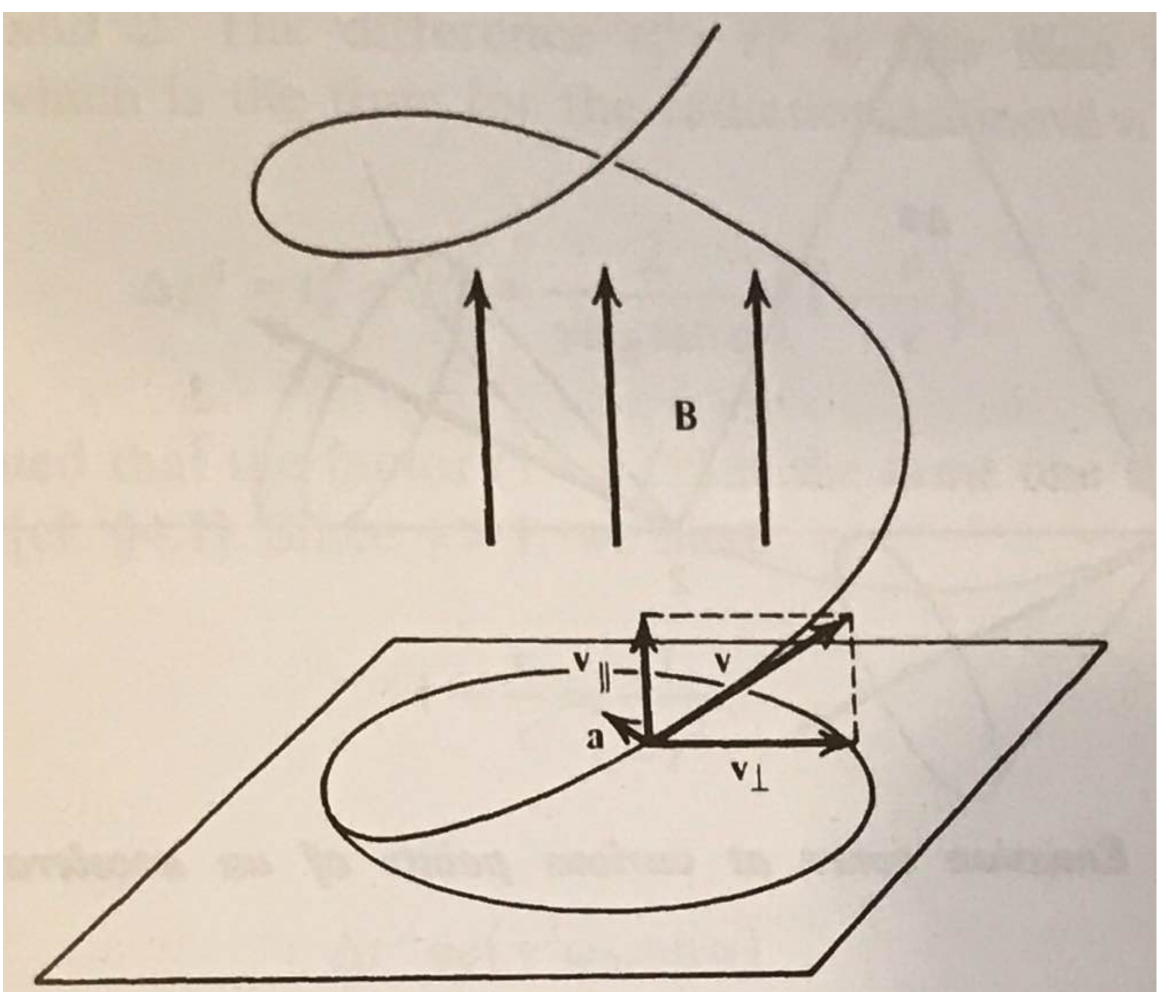
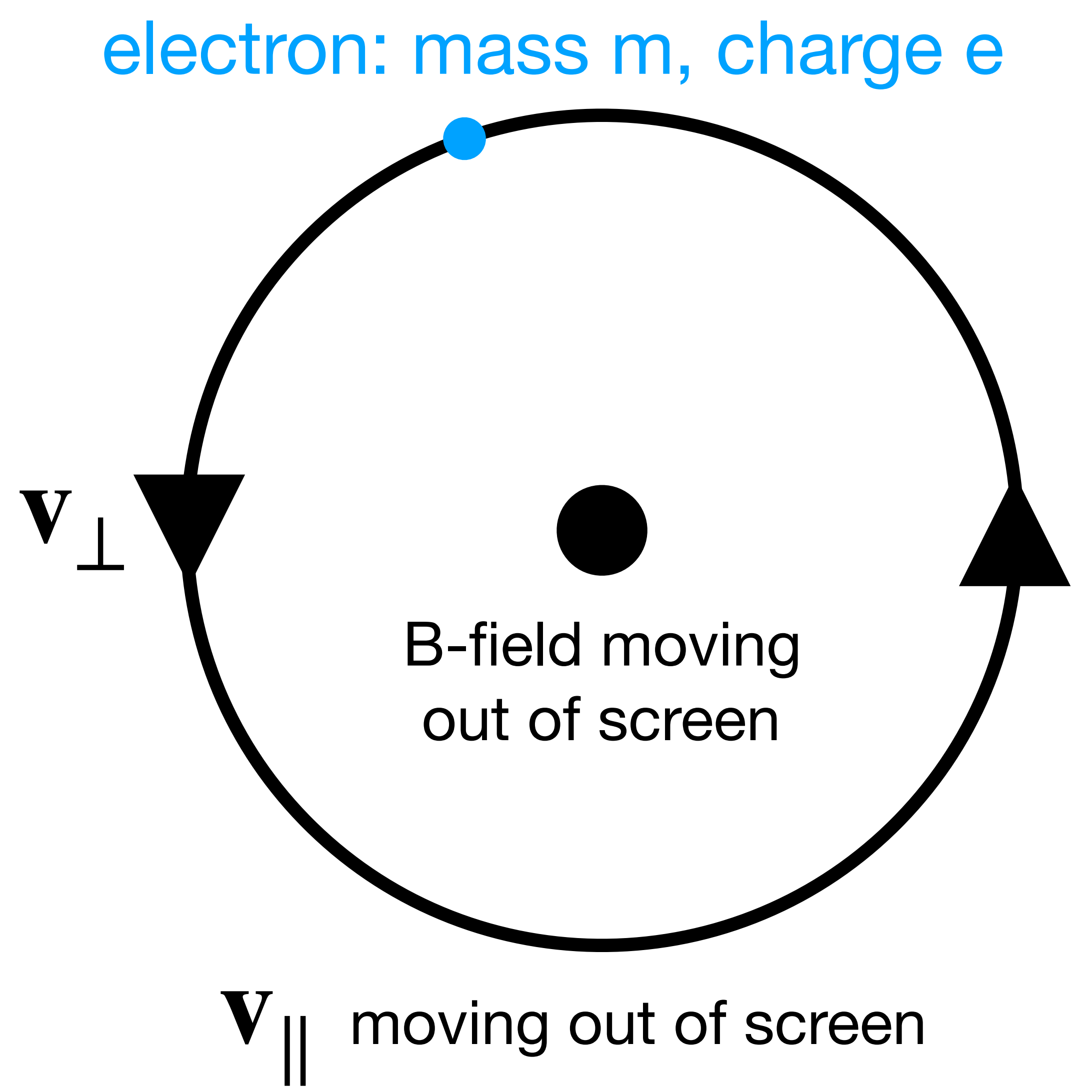
- Motion is <u>*helical*</u>:

$$\mathbf{v}_{\|} = \text{constant}$$ ...velocity parallel to B-field

$$\mathbf{v}_{\perp} = \text{circular}$$ ...velocity perpendicular to B-field





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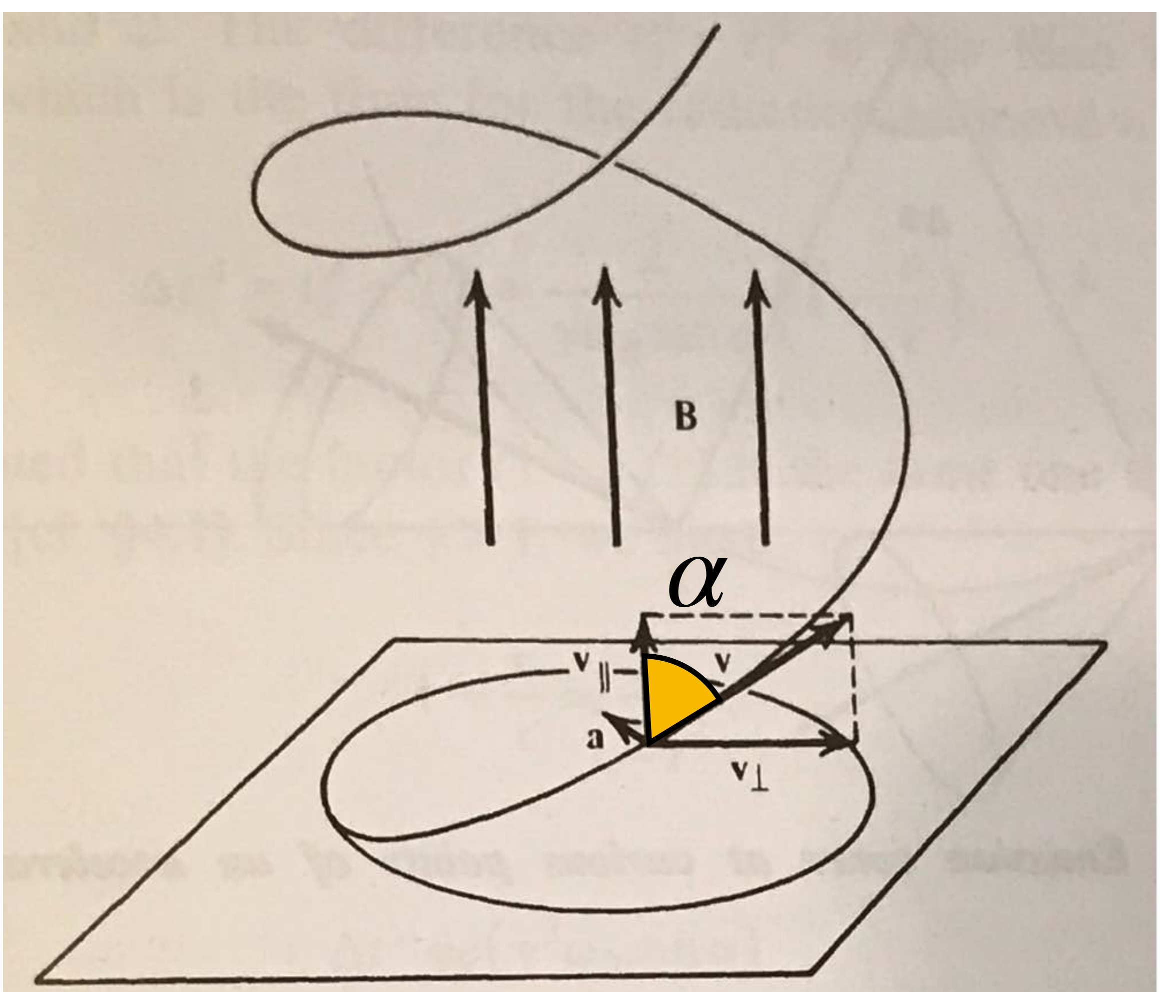
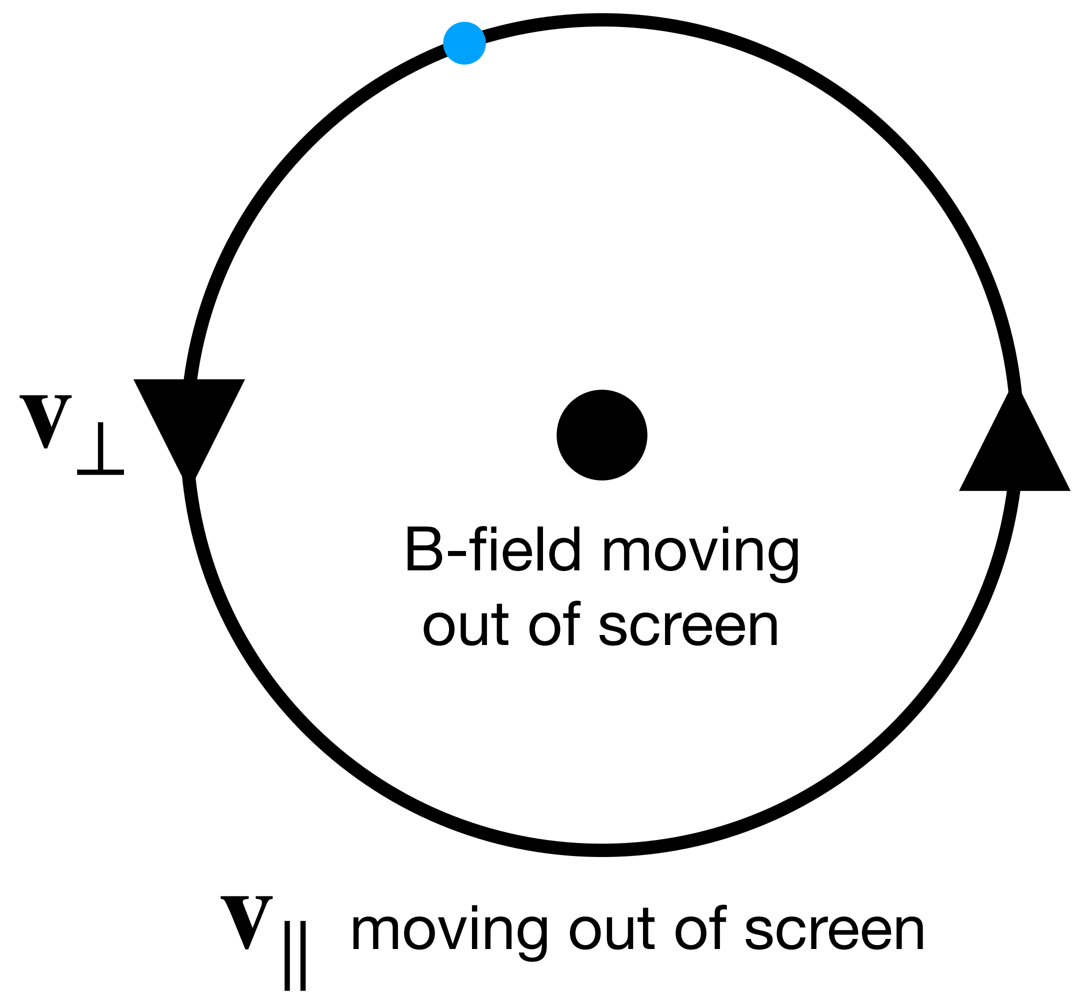
<u>*Pitch angle*</u> = angle between $\mathbf{v}$ and $\mathbf{B}$ = α



electron: mass m, charge e



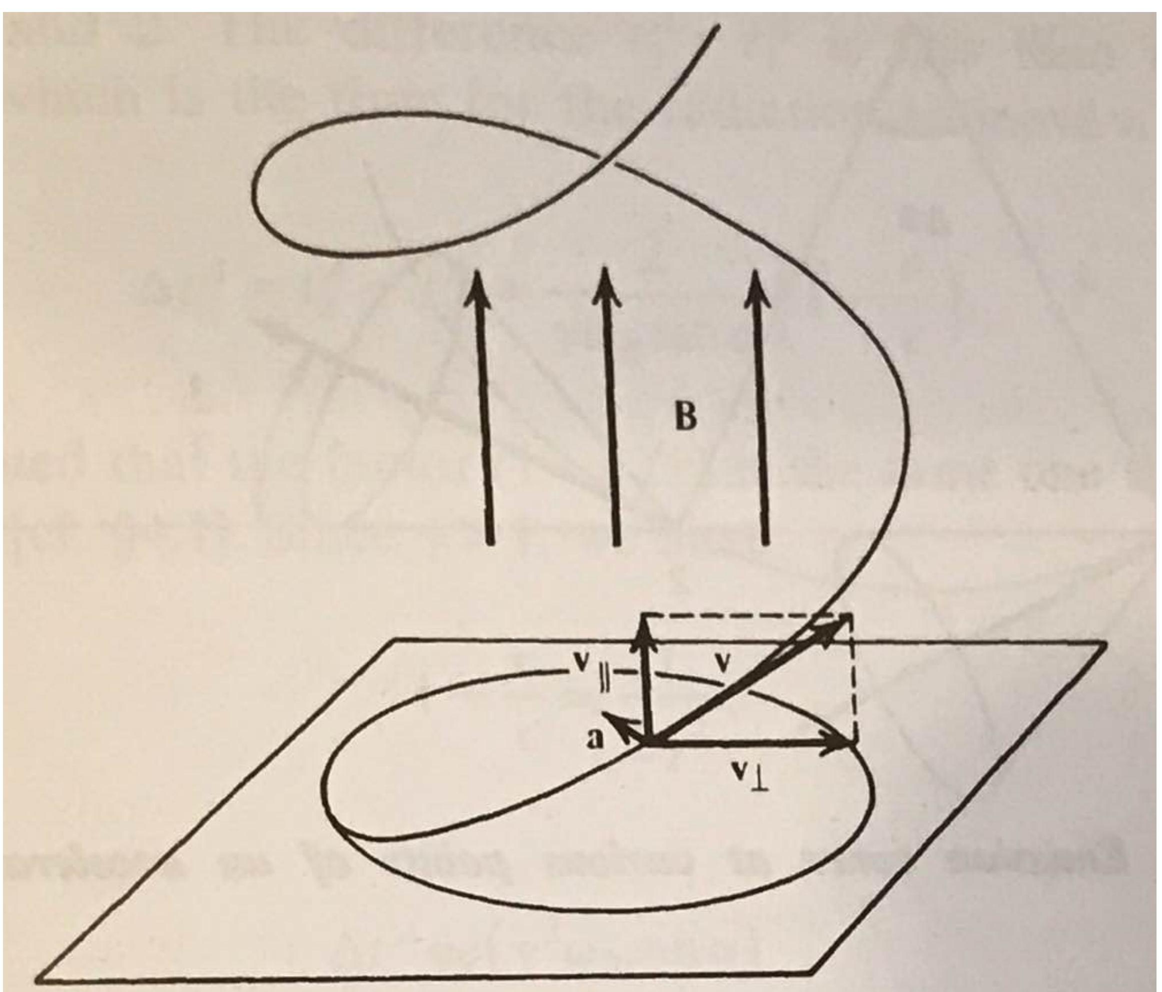
Acceleration of electron

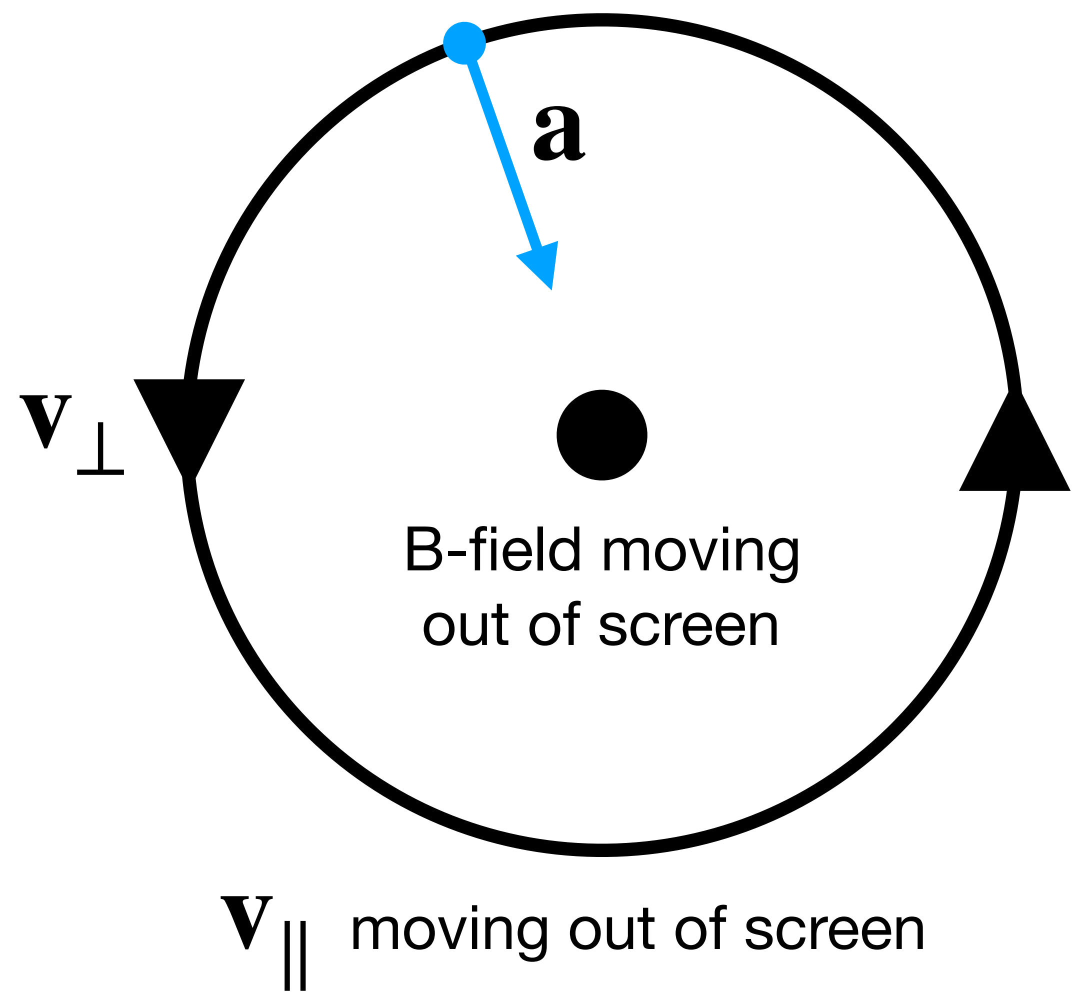
- Lorentz force: $$\mathbf{F} = q(\mathscr{E} + \mathbf{v} \times \mathbf{B}) = -e(\mathscr{E} + \mathbf{v} \times \mathbf{B})$$

Electron has -ve charge

Electric field vector
(E is reserved for electron energy)

Electric charge





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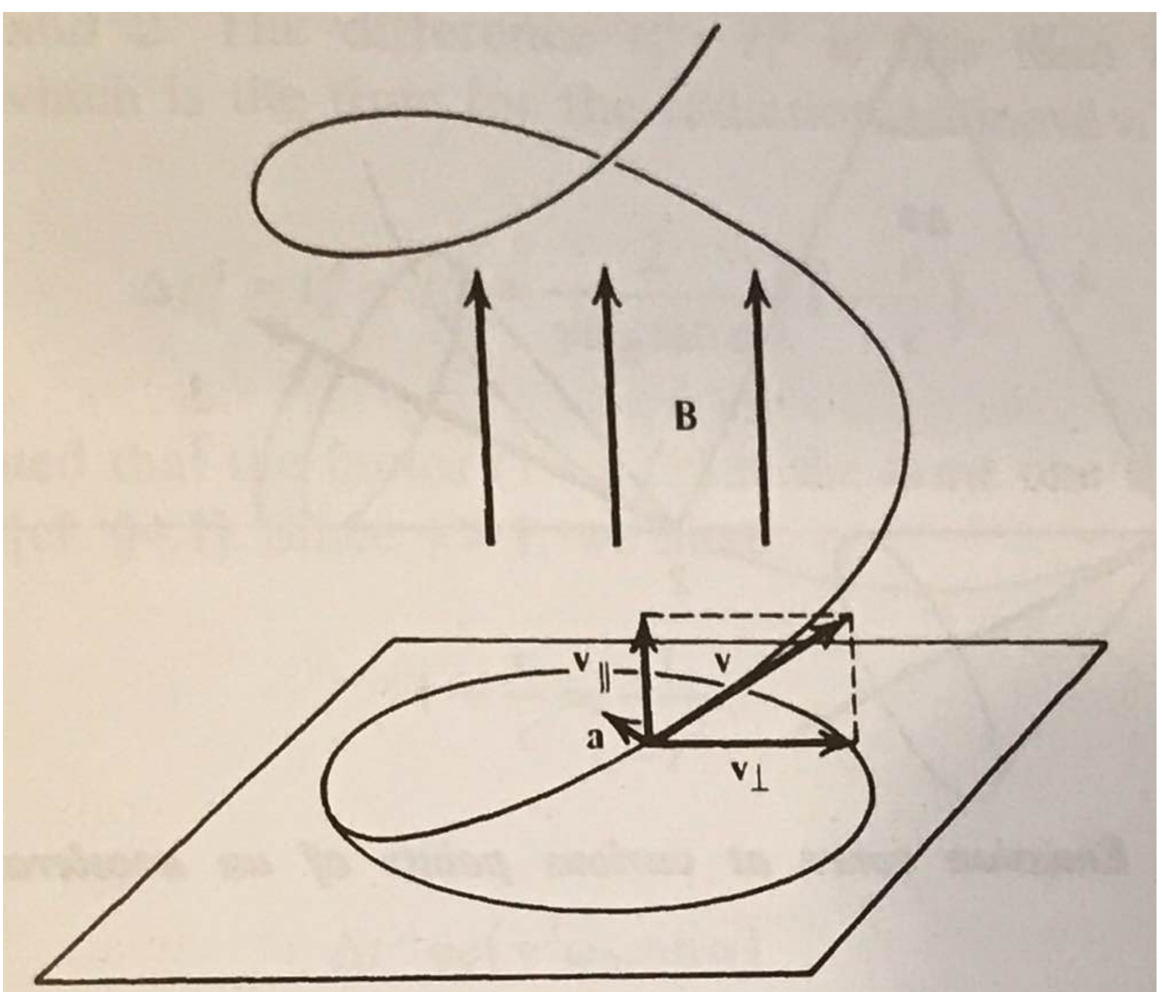

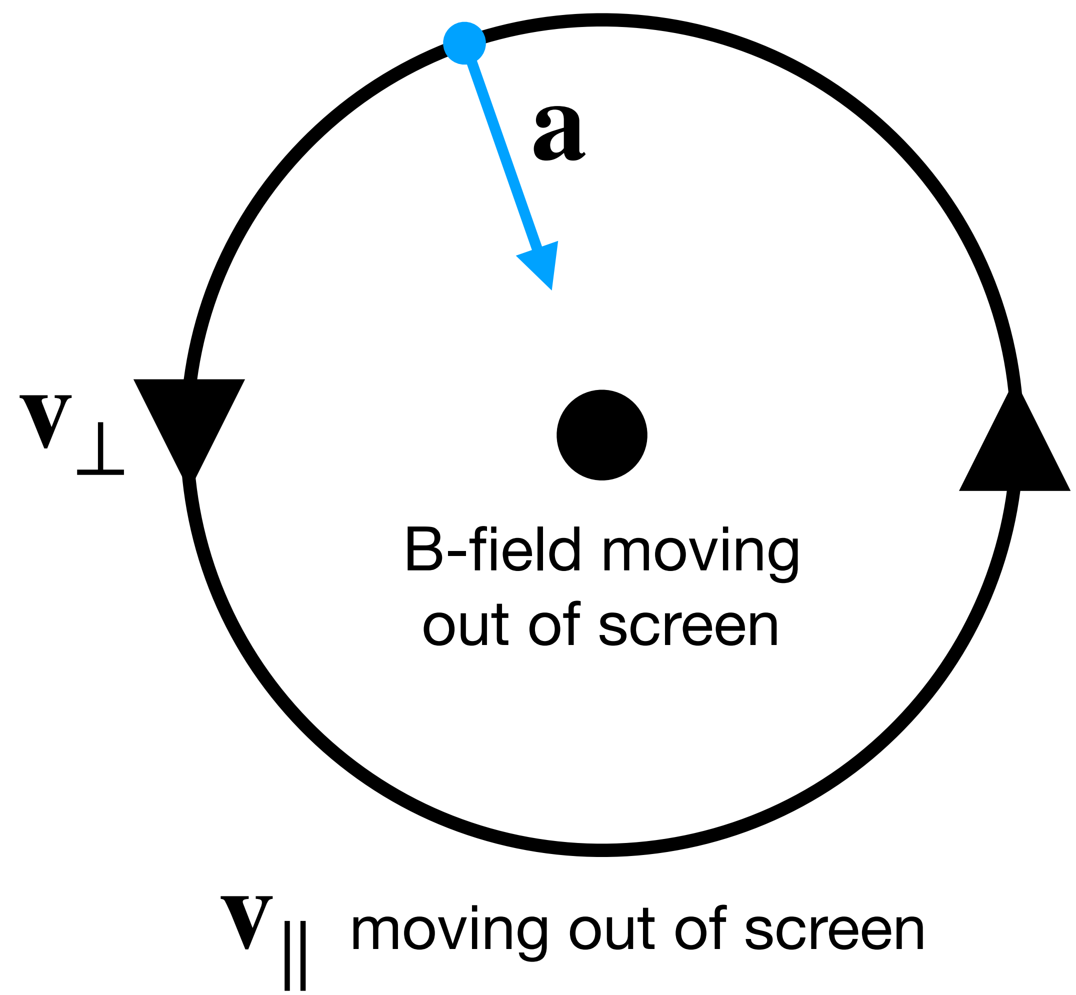

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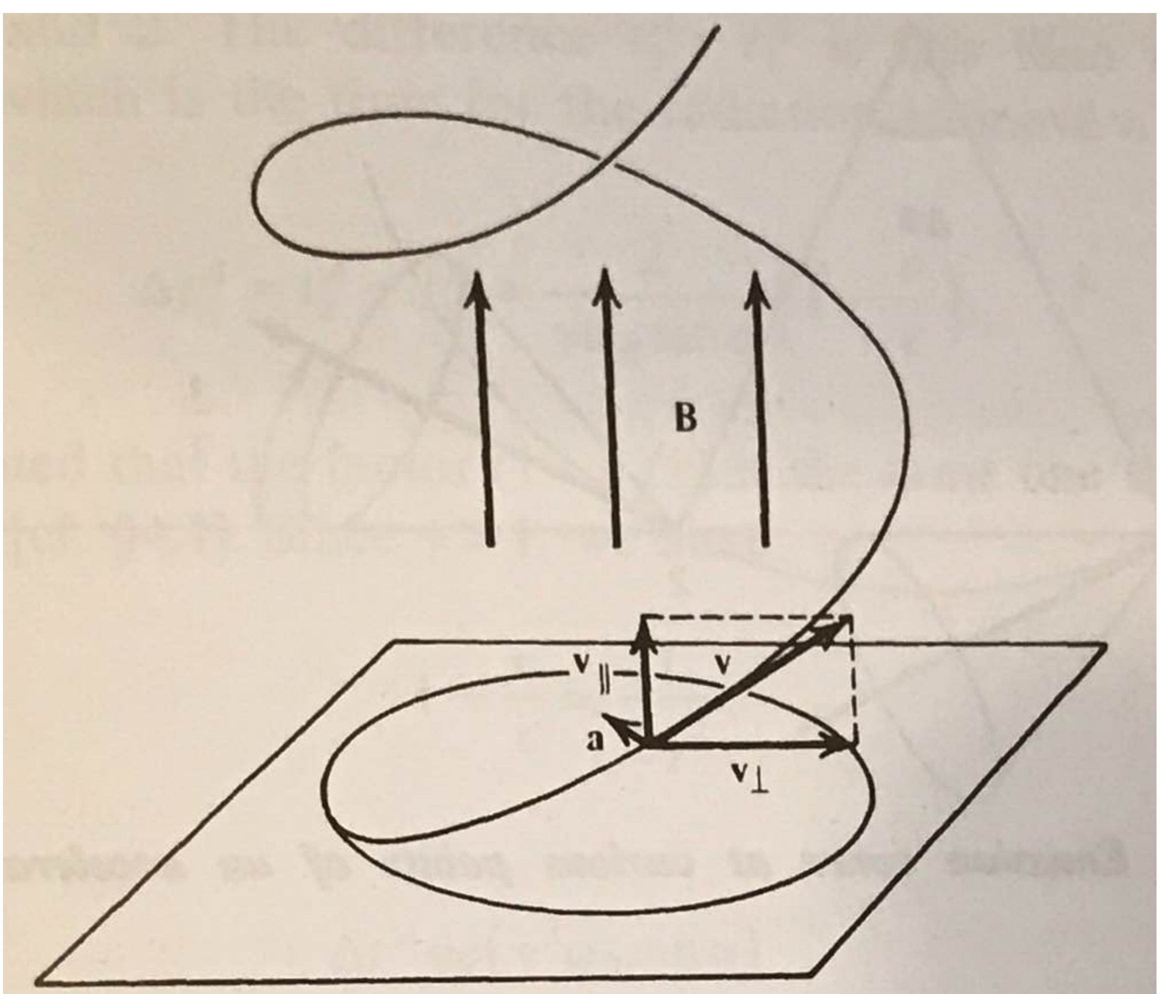

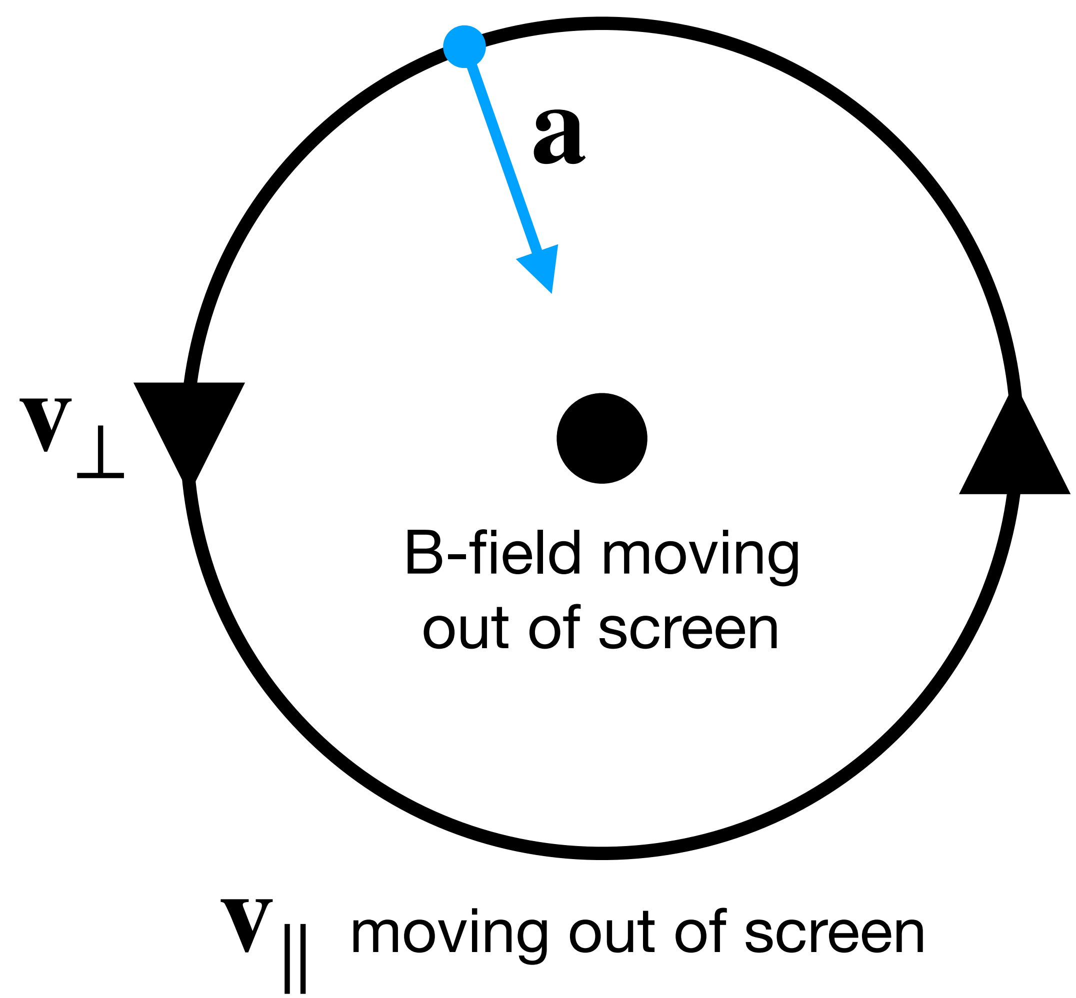

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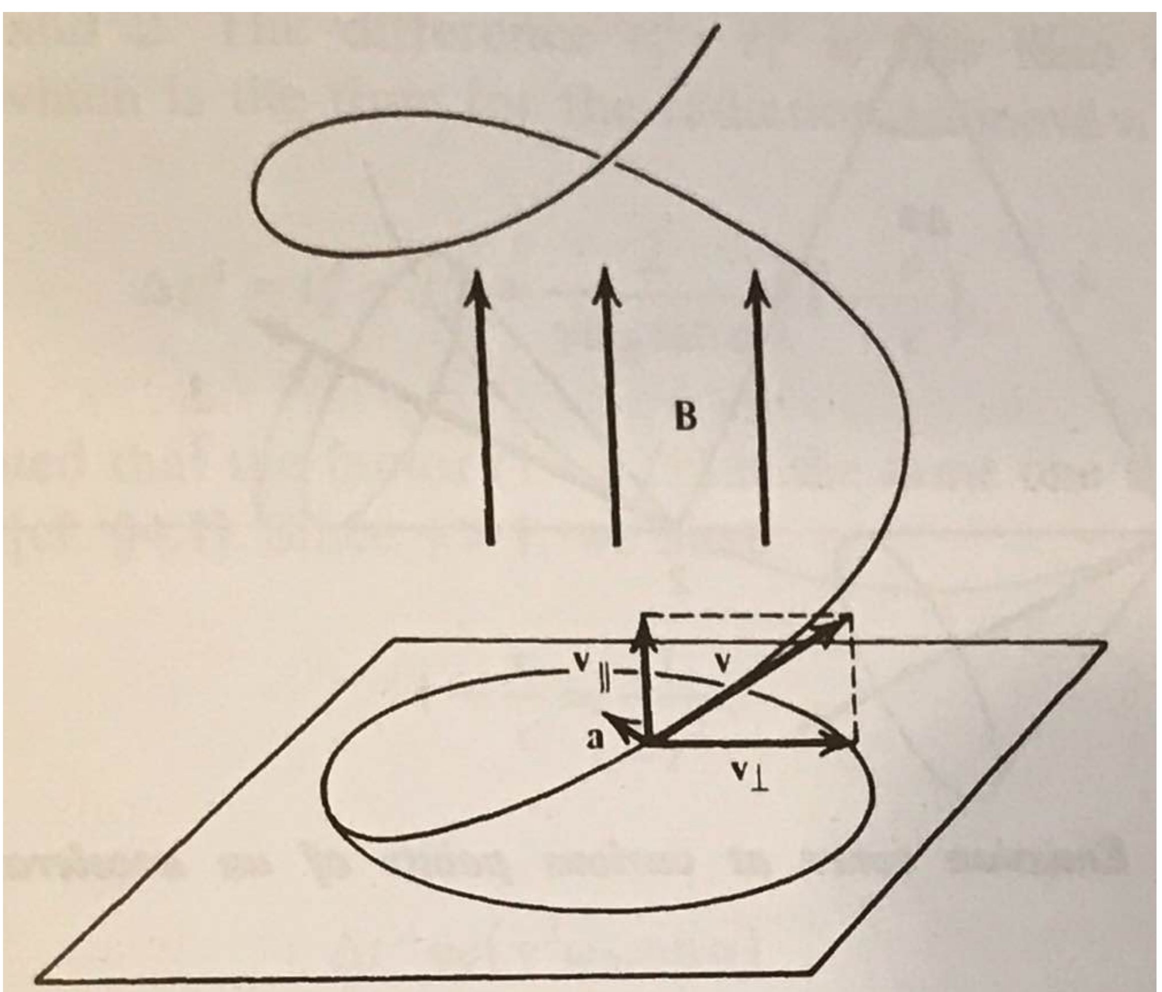
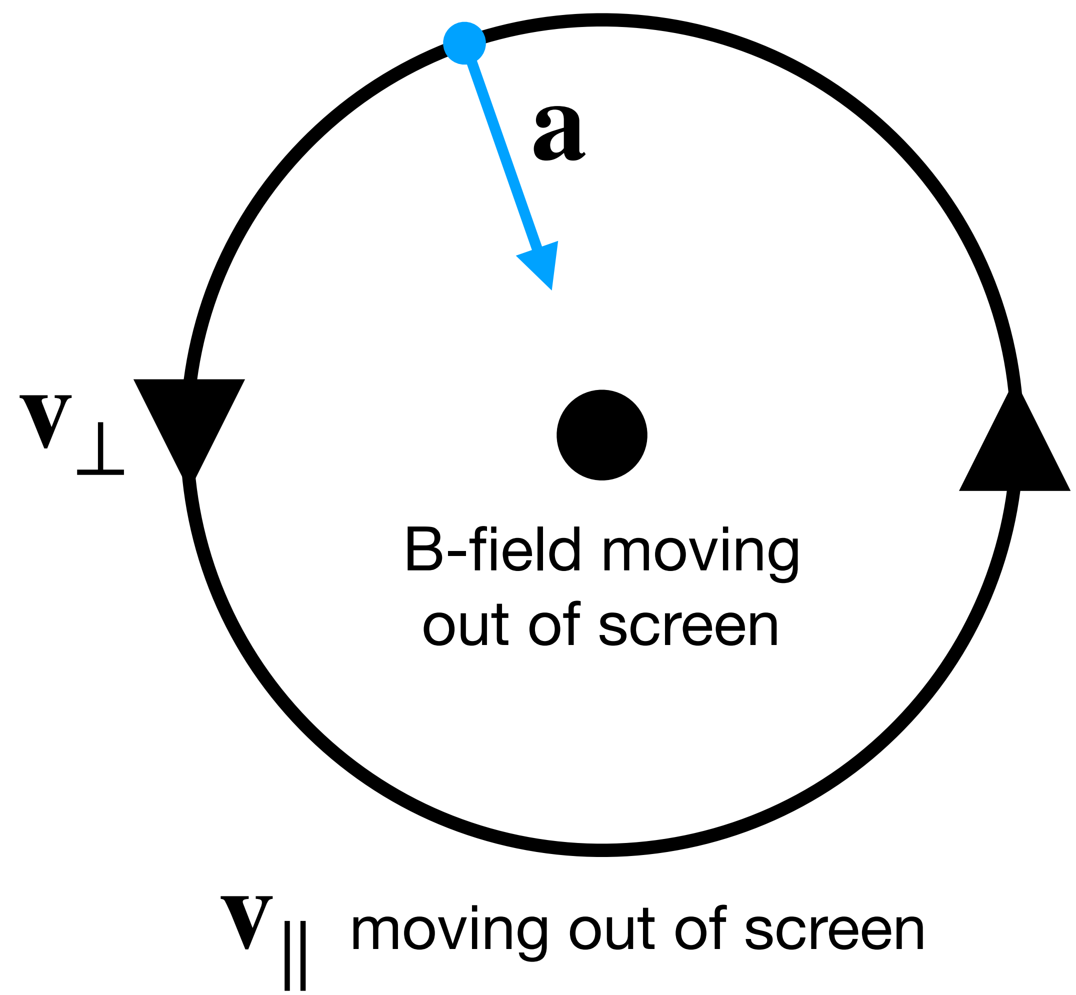
- Therefore acceleration:

$$\mathbf{a} = -\frac{evB\sin\alpha}{\gamma m}\hat{\mathbf{r}}$$

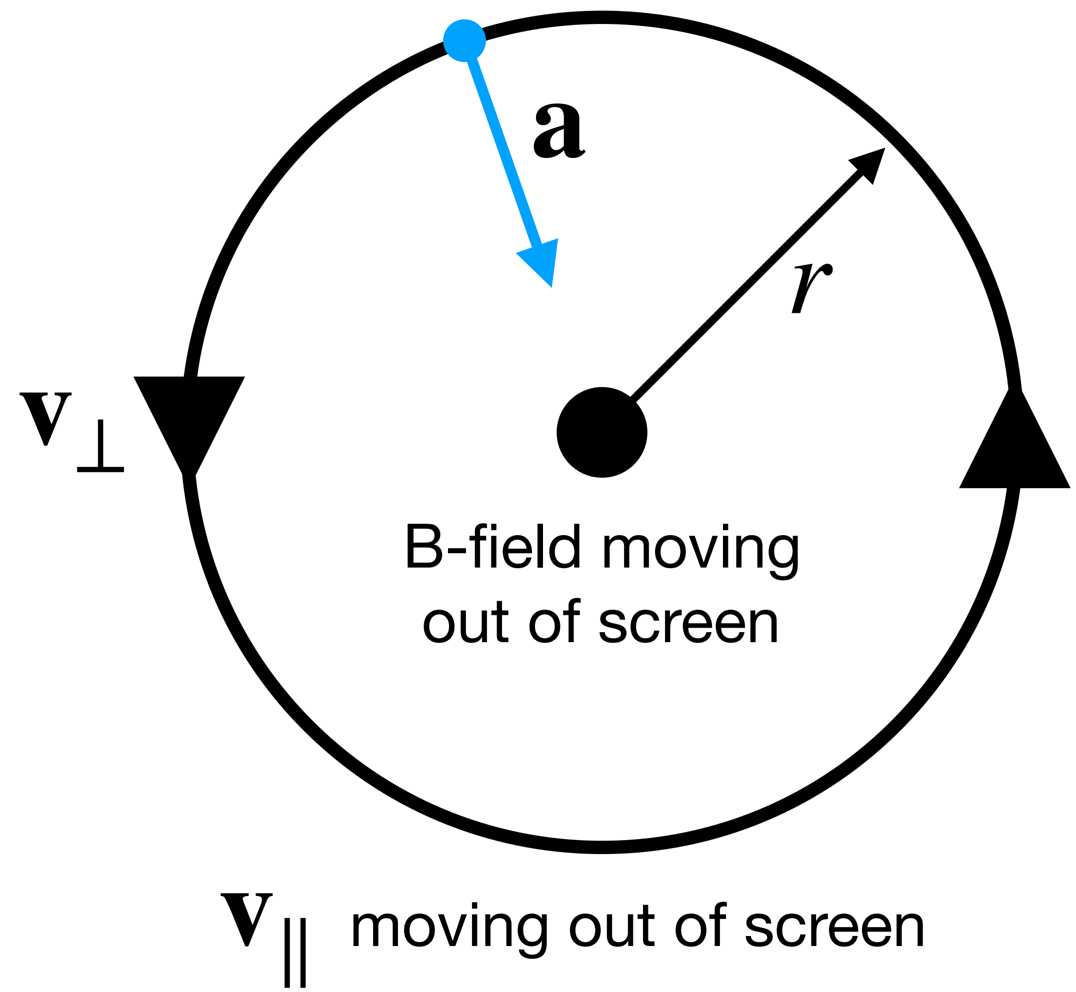




Gyroradius

- Calculate orbital radius of electron by setting a equal to centripetal acceleration:

$$a = \frac{evB\sin\alpha}{\gamma m} = \frac{v_\perp^2}{r} = \frac{v^2\sin^2\alpha}{r}$$



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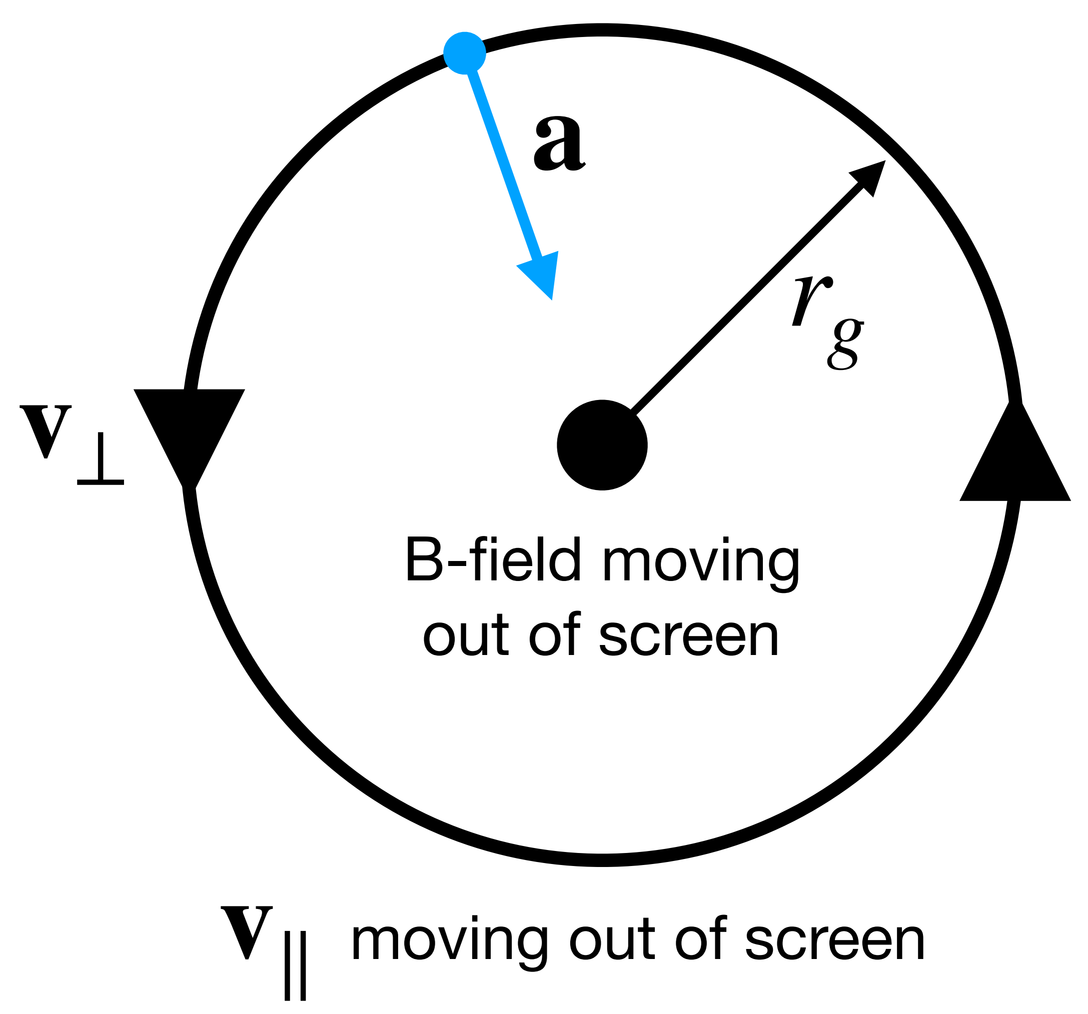

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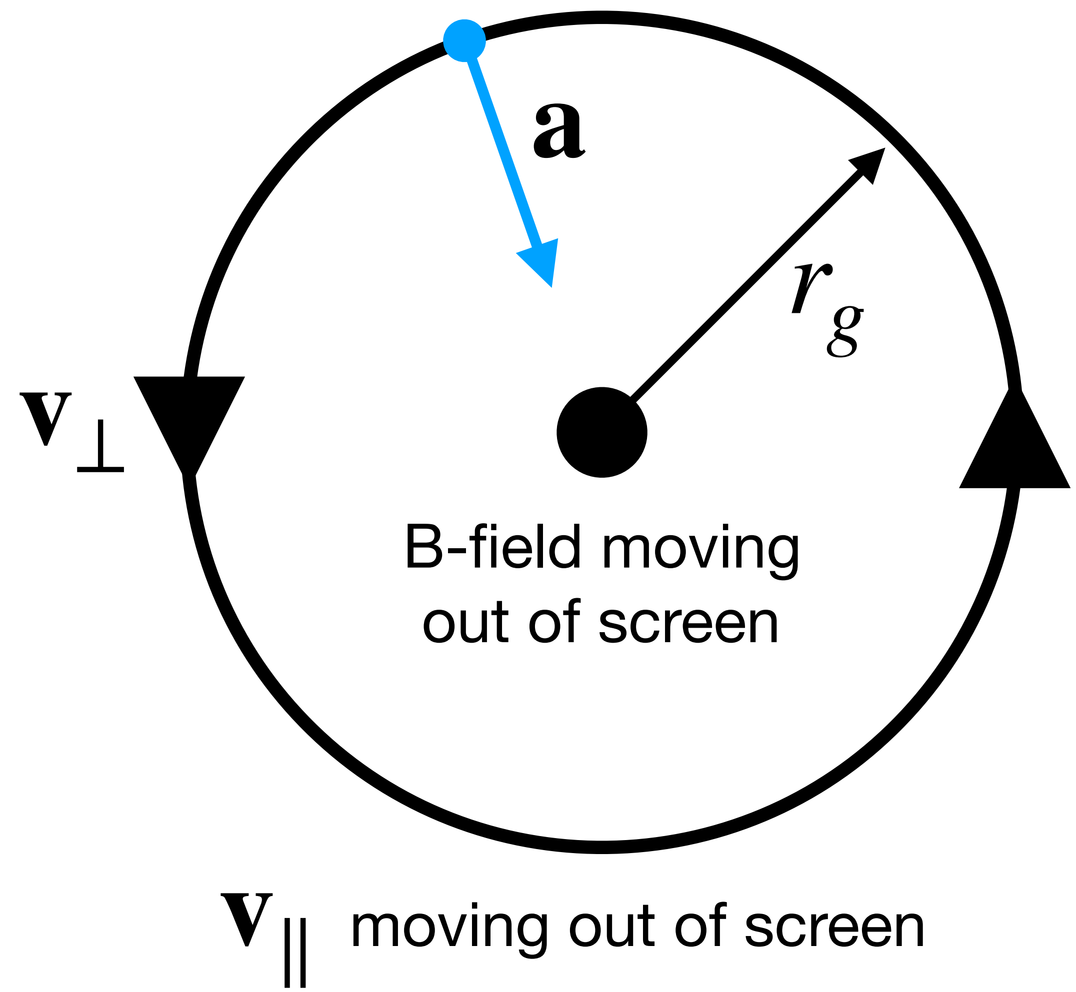

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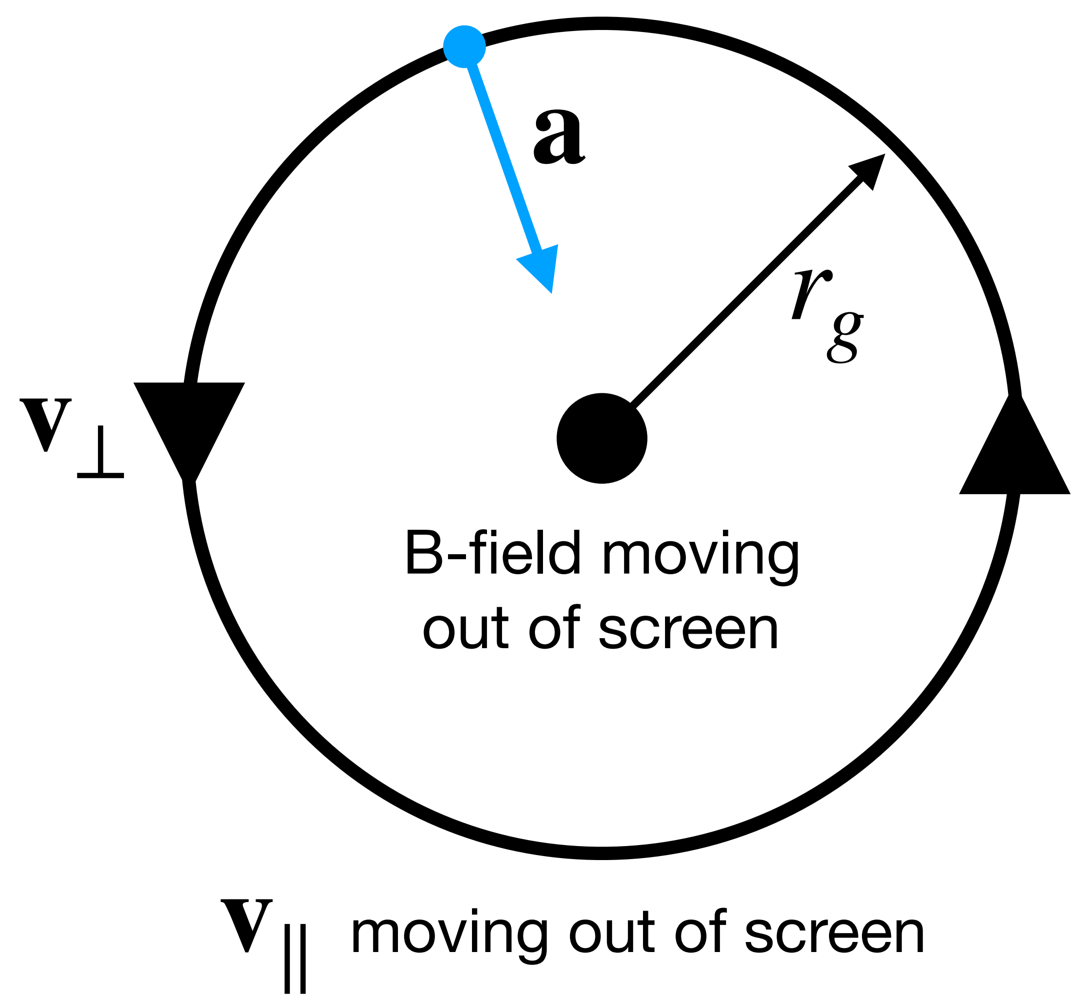

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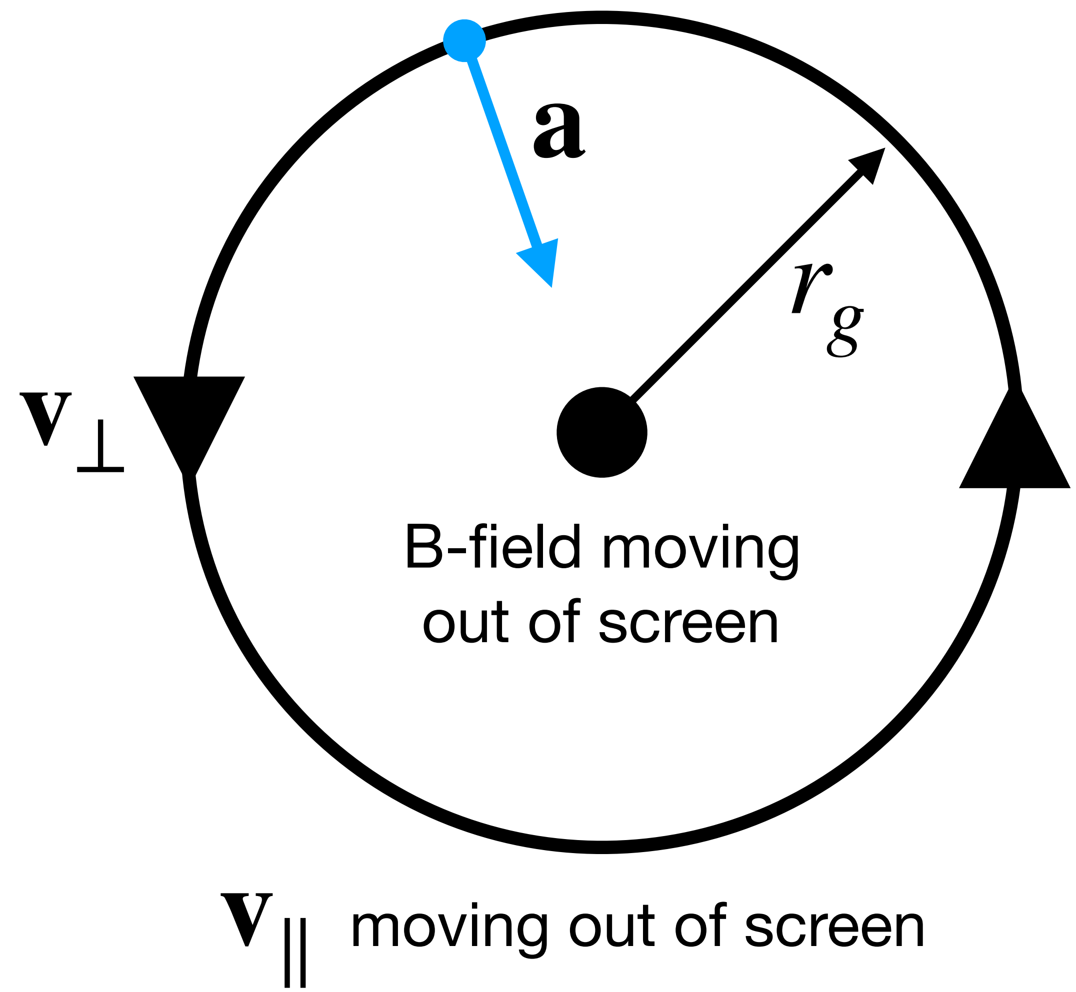
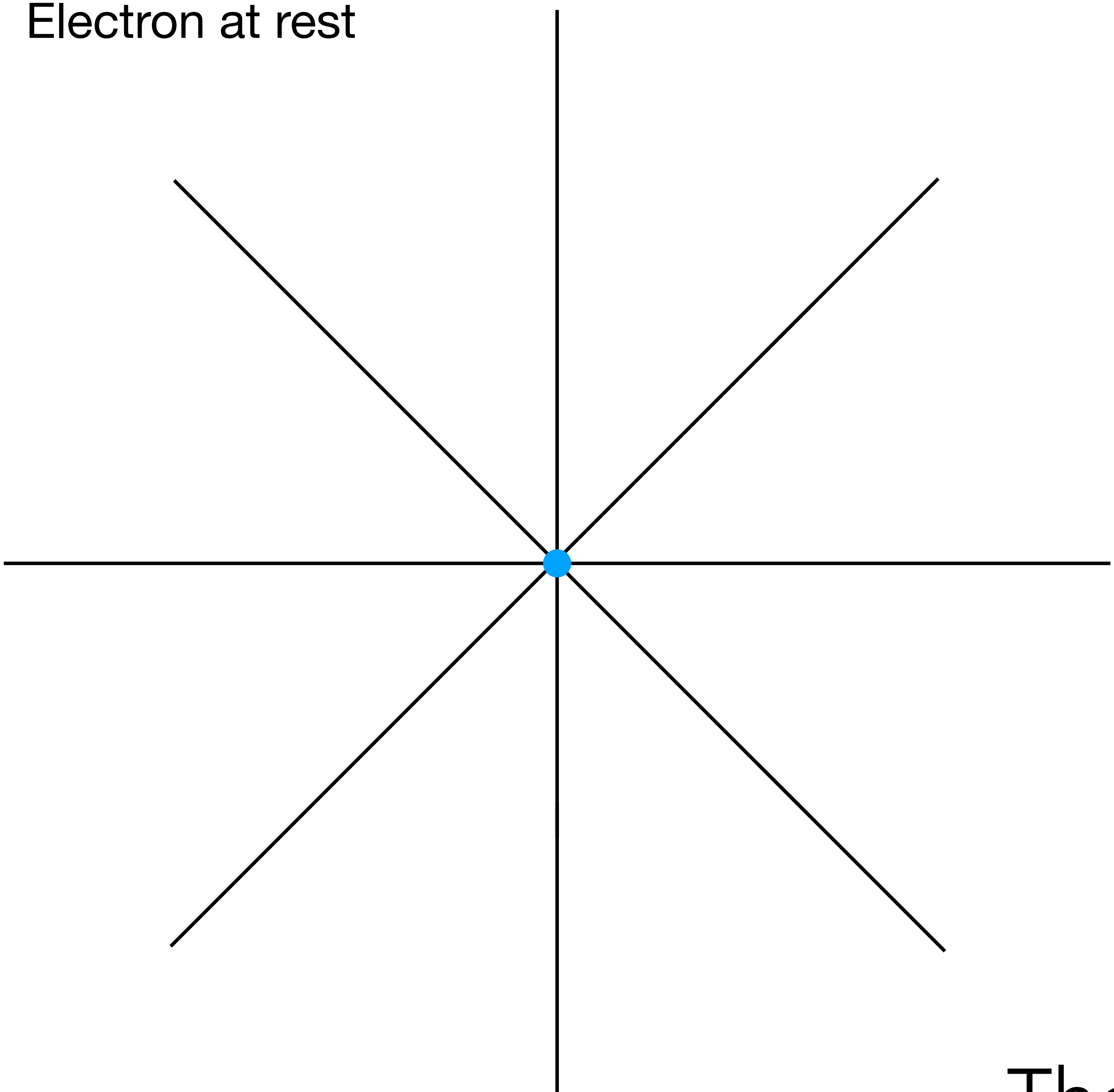
- *Gyroperiod*:

$$t_g = \frac{1}{\nu_g} = \frac{2\pi\gamma m}{eB}$$



Radiation generated

Electron at rest



From Coulomb's law, electric field is:

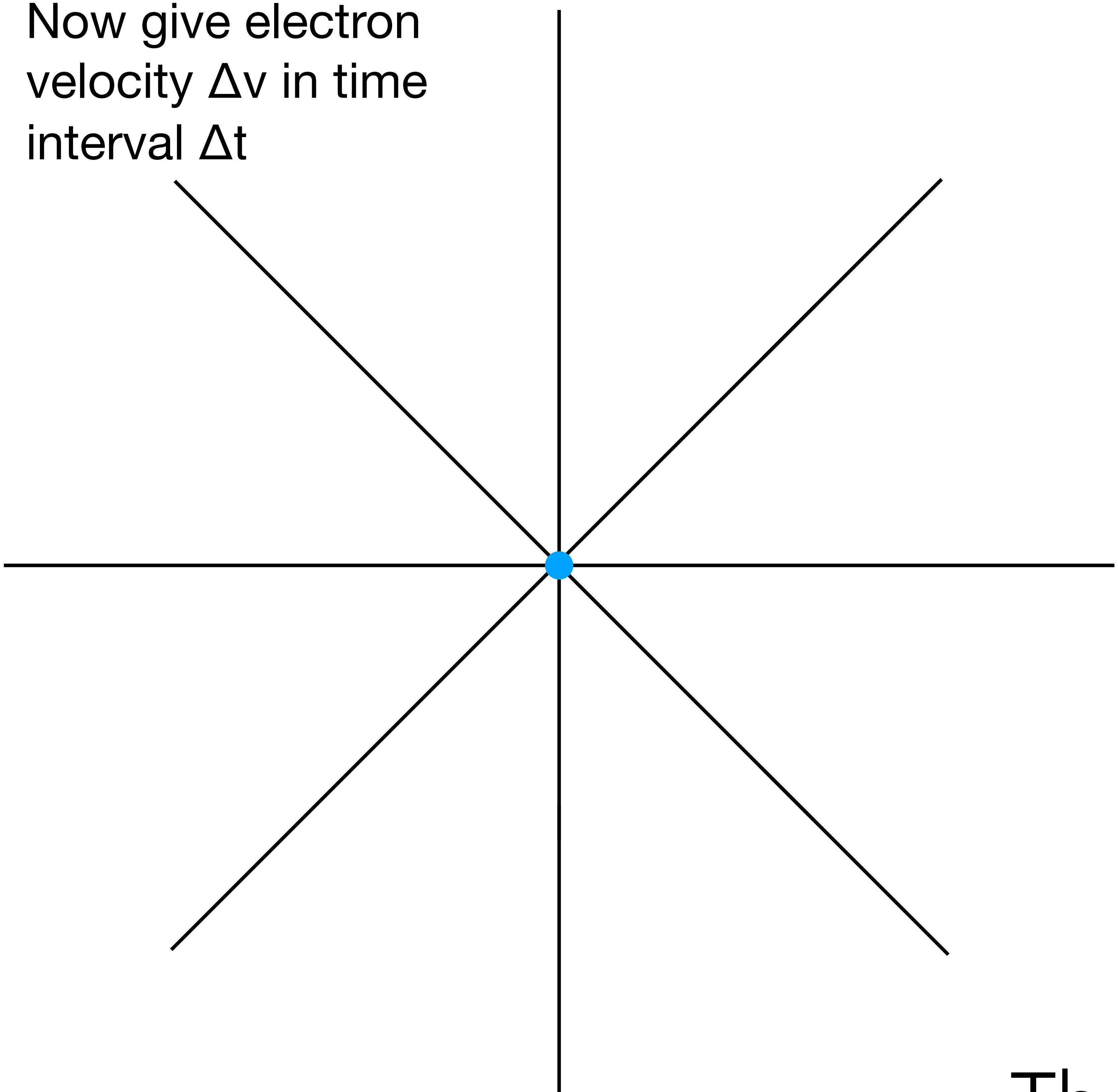
$$\mathscr{E}_r = \frac{e}{4\pi\epsilon_0 r^2}$$

$$\mathscr{E}_\theta = 0$$

Thomson's reasoning

Radiation generated

Now give electron velocity Δv in time interval Δt



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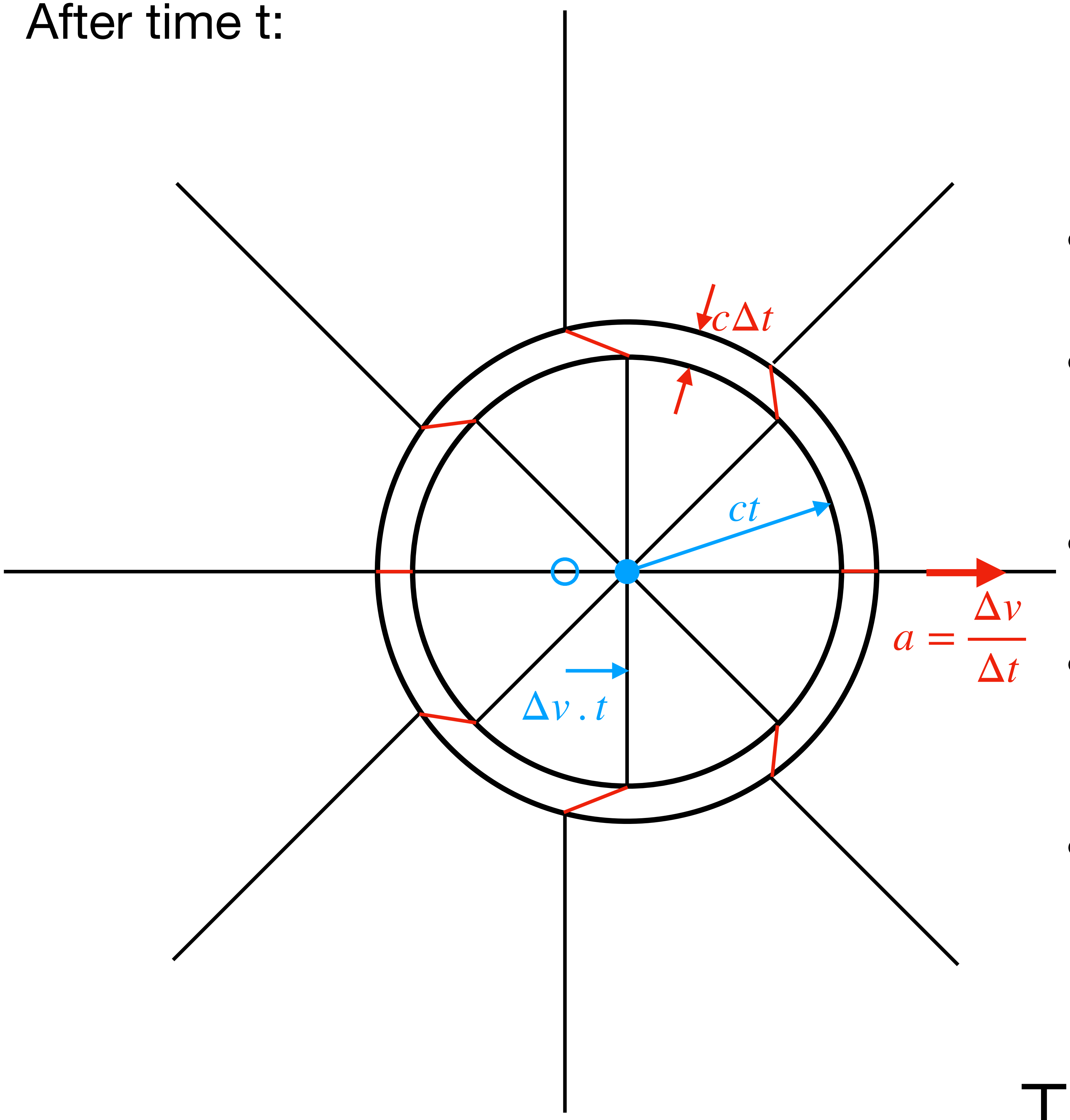
After time t:

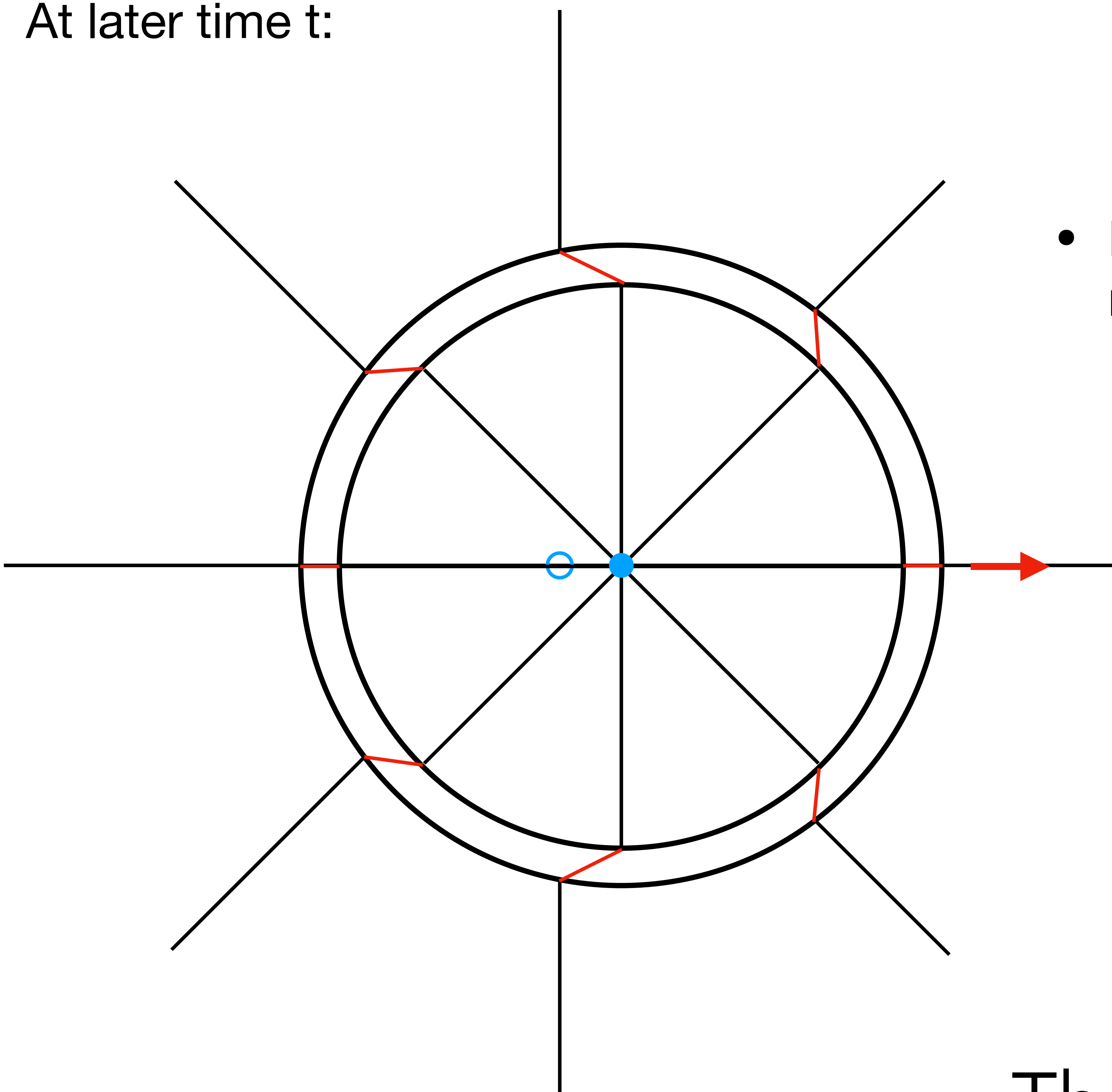


- Electron has travelled distance Δv t
- Electric field is radial centred on electron in sphere of radius ct
- Field outside of this sphere hasn't adjusted yet
- Kink in field in shell of radius cΔt where inner and outer fields join up
- This kink corresponds to non-zero $\mathscr{E}_\theta$

Thomson's reasoning

Radiation generated

At later time t:

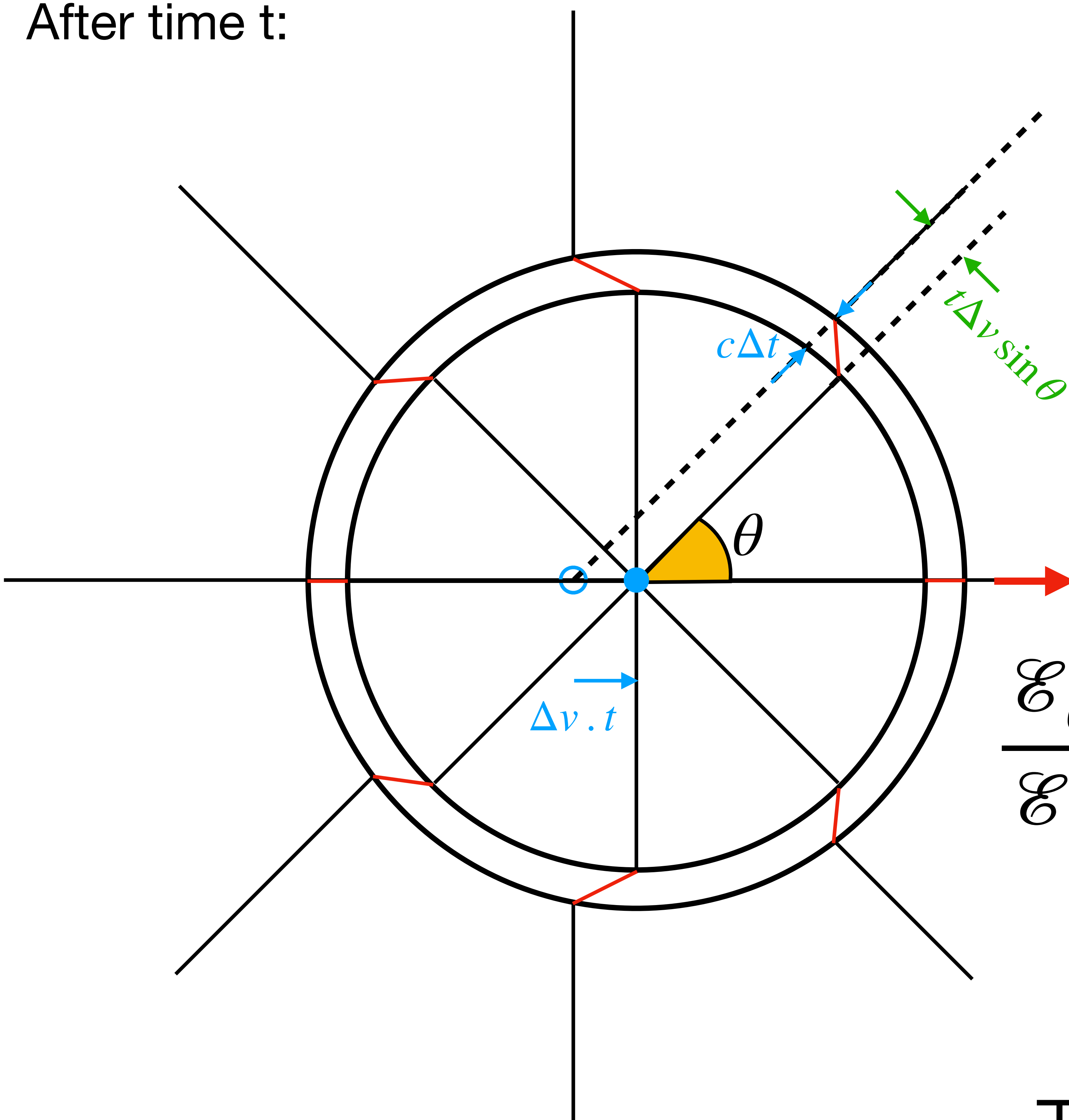


- Pulse of non-zero $\mathscr{E}_\theta$ has moved out

Thomson's reasoning

Radiation generated

After time t:



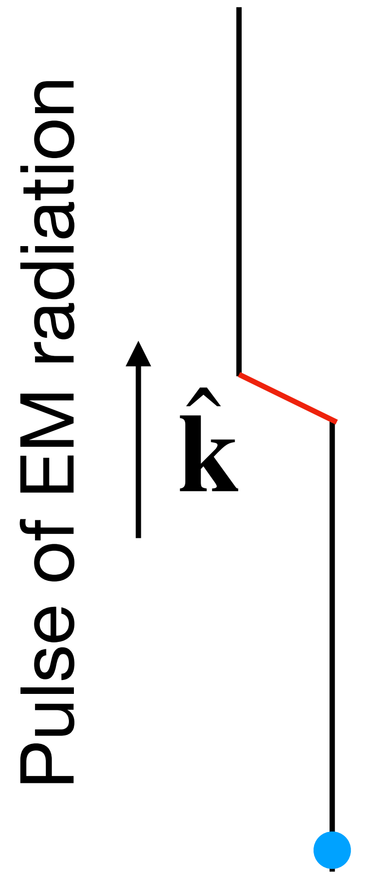
- Pulse of non-zero $\mathscr{E}_\theta$ has moved out
- Pulse strength (in electron rest frame) depends on angle to acceleration θ:

$$\frac{\mathscr{E}_\theta}{\mathscr{E}_r} = \frac{t\Delta v \sin\theta}{c\Delta t} = \frac{ar}{c^2}\sin\theta$$

Thomson's reasoning

Power radiated (non-relativistic)

$$\mathscr{E}_{\text{pulse}} = \mathscr{E}_{\theta} \qquad \mathbf{B}_{\text{pulse}} = \frac{1}{c}\hat{\mathbf{k}} \times \mathscr{E}_{\text{pulse}} \qquad c^2 = \frac{1}{\epsilon_0 \mu_0}$$



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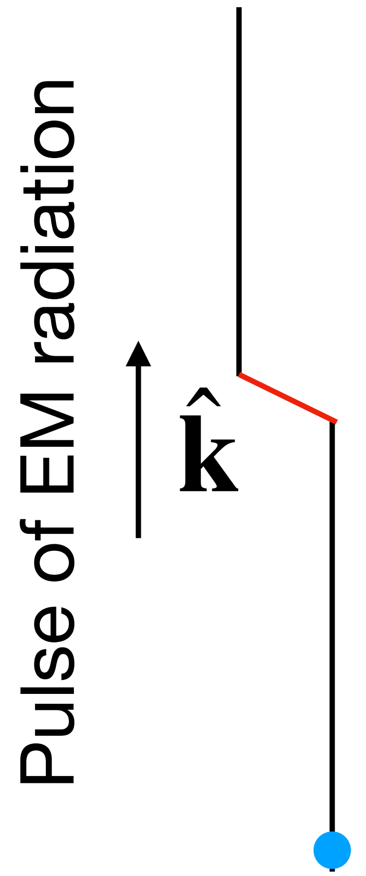

Energy flow in pulse / time / area at distance r = modulus of Poynting vector:

$$|\mathbf{S}| = \frac{1}{\mu_0} |\mathscr{E}_{\text{pulse}} \times \mathbf{B}_{\text{pulse}}| = \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \mathscr{E}_\theta^2$$

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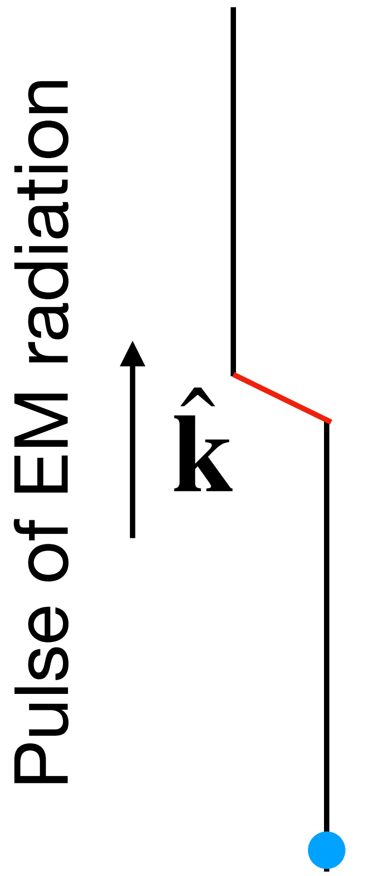

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$$\frac{\mathcal{E}_{\theta}}{\mathcal{E}_r} = \frac{ar}{c^2} \sin\theta \quad \Longrightarrow$$

$$\mathcal{E}_r = \frac{e}{4\pi\epsilon_0 r^2} \quad \Longrightarrow$$

$$S = \frac{e^2 a^2 \sin^2\theta}{(4\pi)^2 \epsilon_0 c^3 r^2}$$

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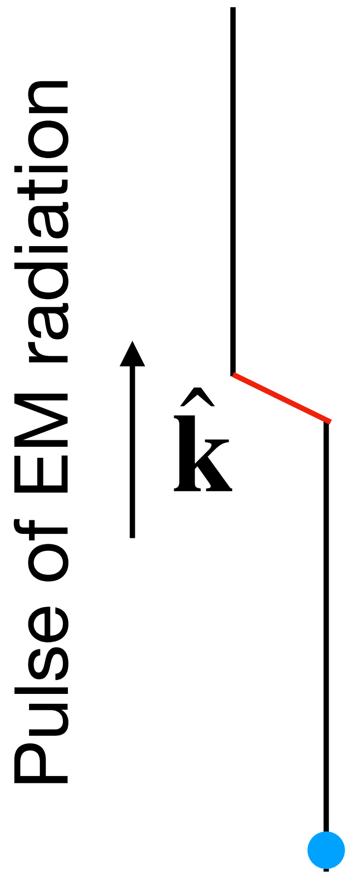

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Therefore power radiated into full sphere (remember dA = r 2π sinθ dθ):

$$P = \frac{e^2 a^2}{6\pi\epsilon_0 c^3}$$

Power radiated

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- We want to deal with ultra-relativistic electrons, so need relativistic limit! Luckily, can use non-relativistic formula in the instantaneous electron rest frame S' and then use Lorentz invariance of dE/dt to trivially move back to the observer's frame S (in which the B-field is at rest)

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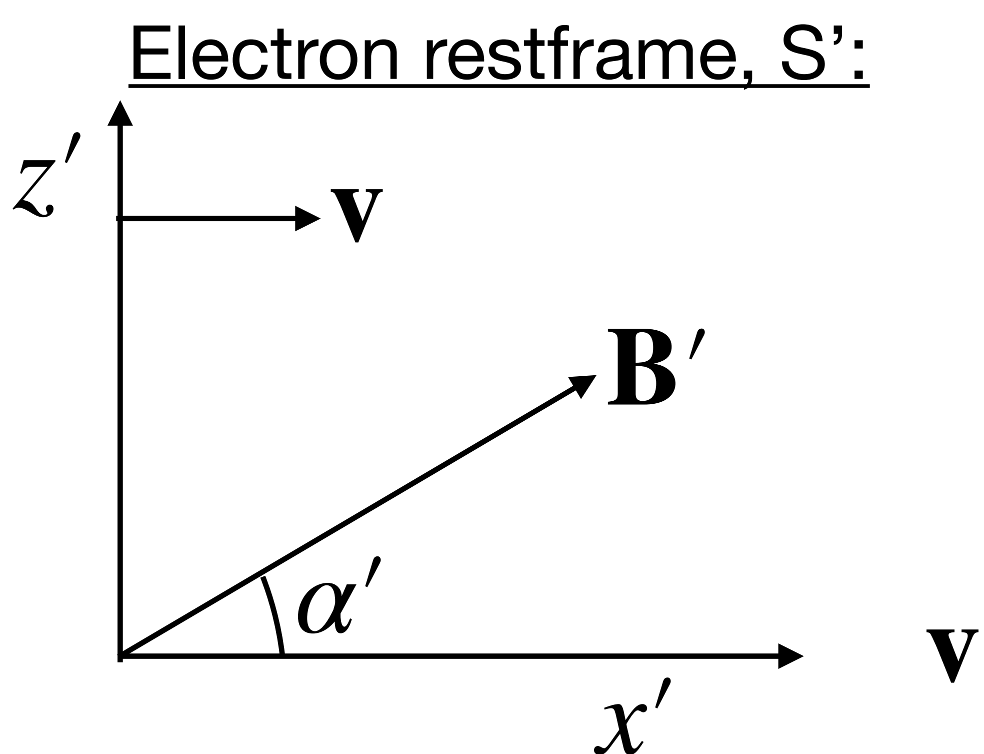

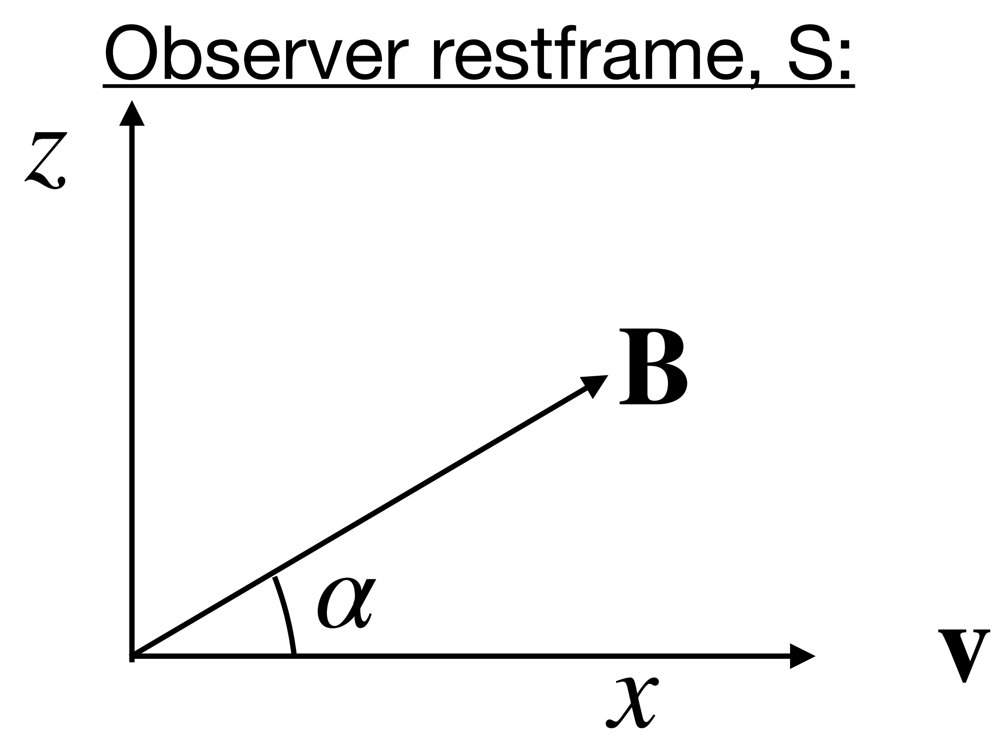

Power radiated

- In S frame: $B_x = B\cos\alpha;\;\; B_y = 0;\;\; B_z = B\sin\alpha$

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- Lorentz transforms to get electric field in S':

$$\mathscr{E}'_x = \mathscr{E}_x$$
$$\mathscr{E}'_y = \gamma(\mathscr{E}_y - vB_z)$$
$$\mathscr{E}'_z = \gamma(\mathscr{E}_z - vB_y)$$

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Power radiated

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- Re-arrange:

$$c^2 = \frac{1}{\epsilon_0\mu_0} \qquad U_{\mathrm{mag}} = \frac{B^2}{2\mu_0}$$ = energy density of magnetic field

$$\sigma_T = \frac{e^4}{6\pi\epsilon_0^2 c^4 m^2}$$ = Thomson cross-section

$$\implies \qquad P = 2\sigma_T c U_{\mathrm{mag}} \left(\frac{v}{c}\right)^2 \gamma^2 \sin^2\alpha$$

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$$c^2 = \frac{1}{\epsilon_0\mu_0} \qquad U_{\mathrm{mag}} = \frac{B^2}{2\mu_0}$$ = energy density of magnetic field

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$$\implies \qquad P = 2\sigma_T c U_{\mathrm{mag}} \left(\frac{v}{c}\right)^2 \gamma^2 \sin^2\alpha$$

i.e. Tighter helixes with the same v radiate more (because more of velocity is in circular motion)

Power radiated

$$P = 2\sigma_T c U_{\text{mag}} \left(\frac{v}{c}\right)^2 \gamma^2 \sin^2 \alpha$$

- Average over isotropic distribution of pitch angles:

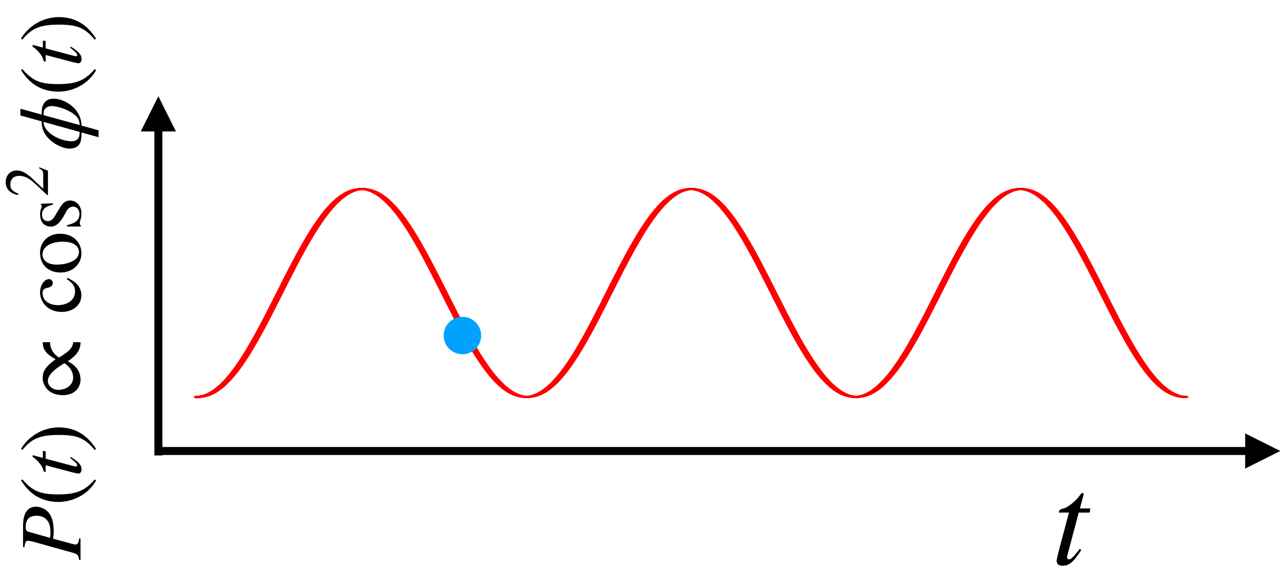
$$\therefore \langle P \rangle = \frac{4}{3}\sigma_T c U_{\text{mag}} \left(\frac{v}{c}\right)^2 \gamma^2$$

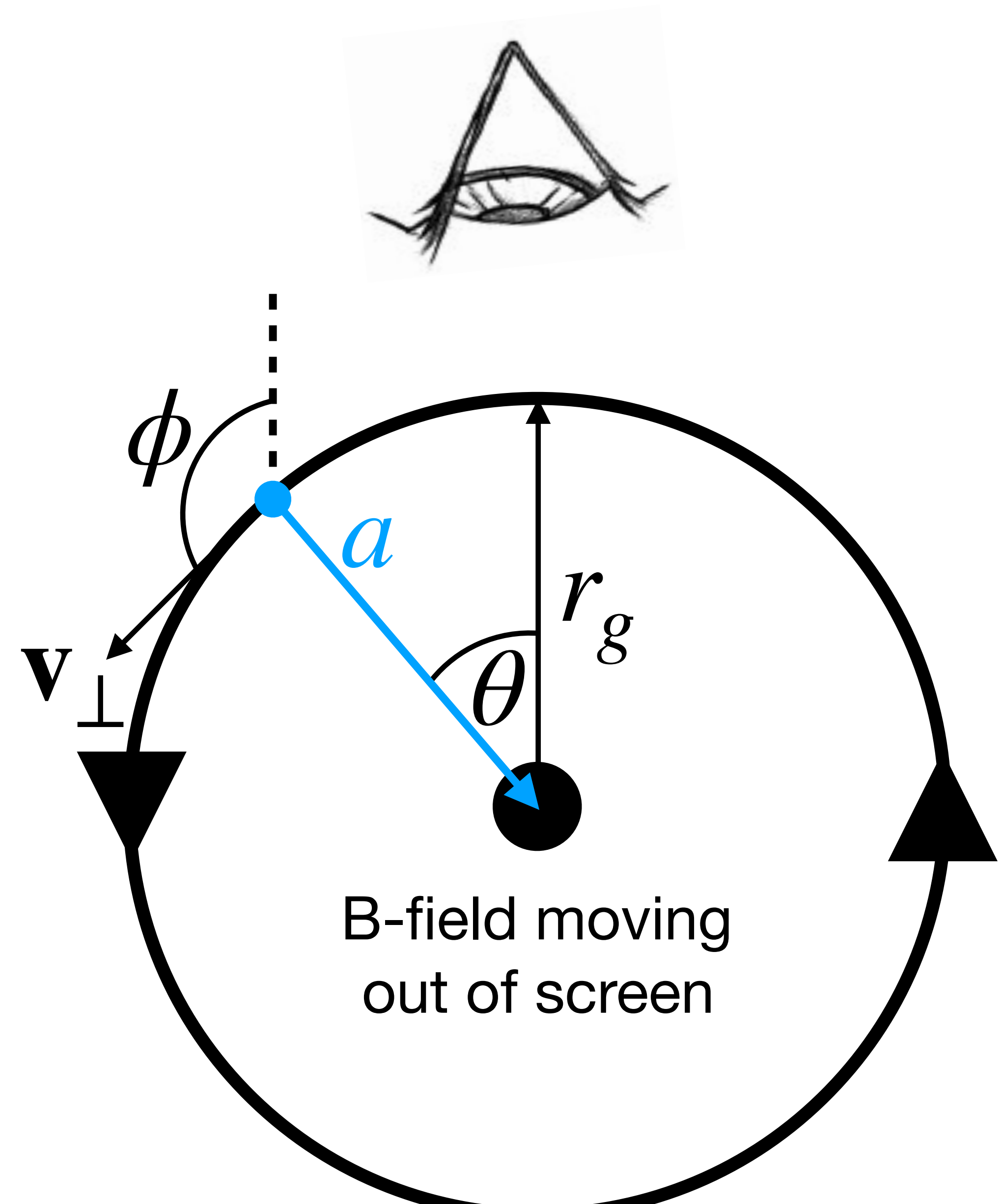
i.e. For a population of electrons travelling in random initial directions but all with the same speed (and therefore the same Lorentz factor and same energy).

Spectrum

γ=1: gyroradiation

Linearly polarised





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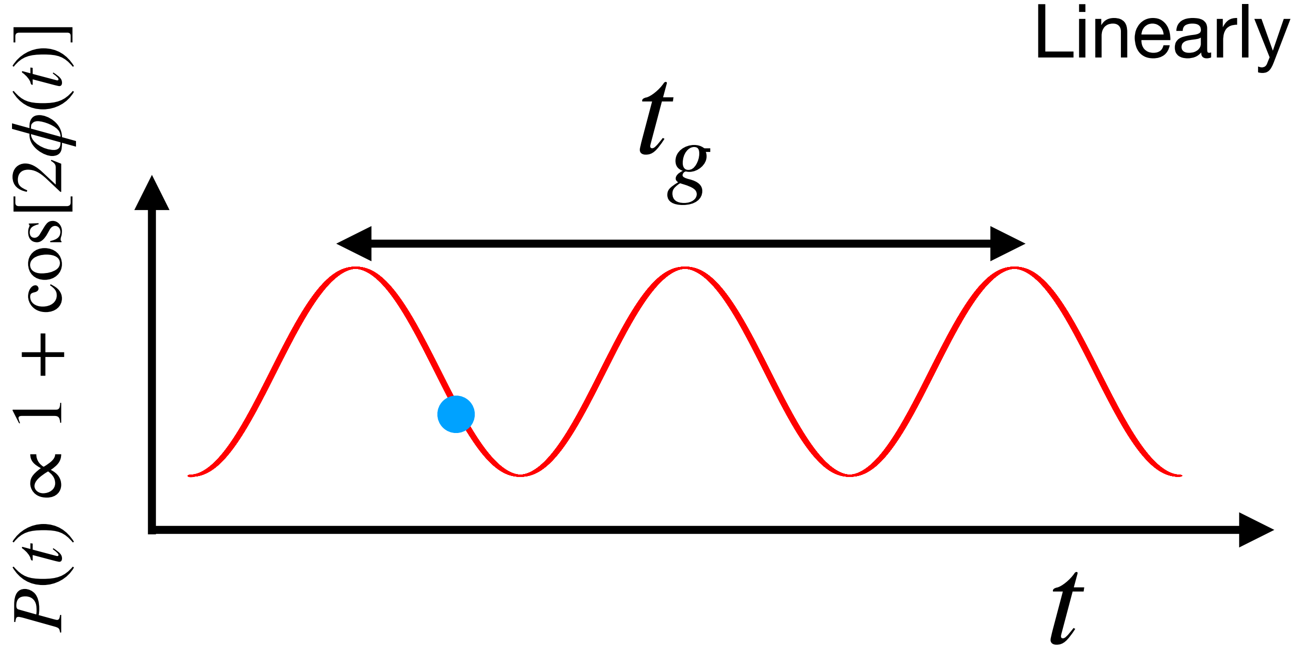

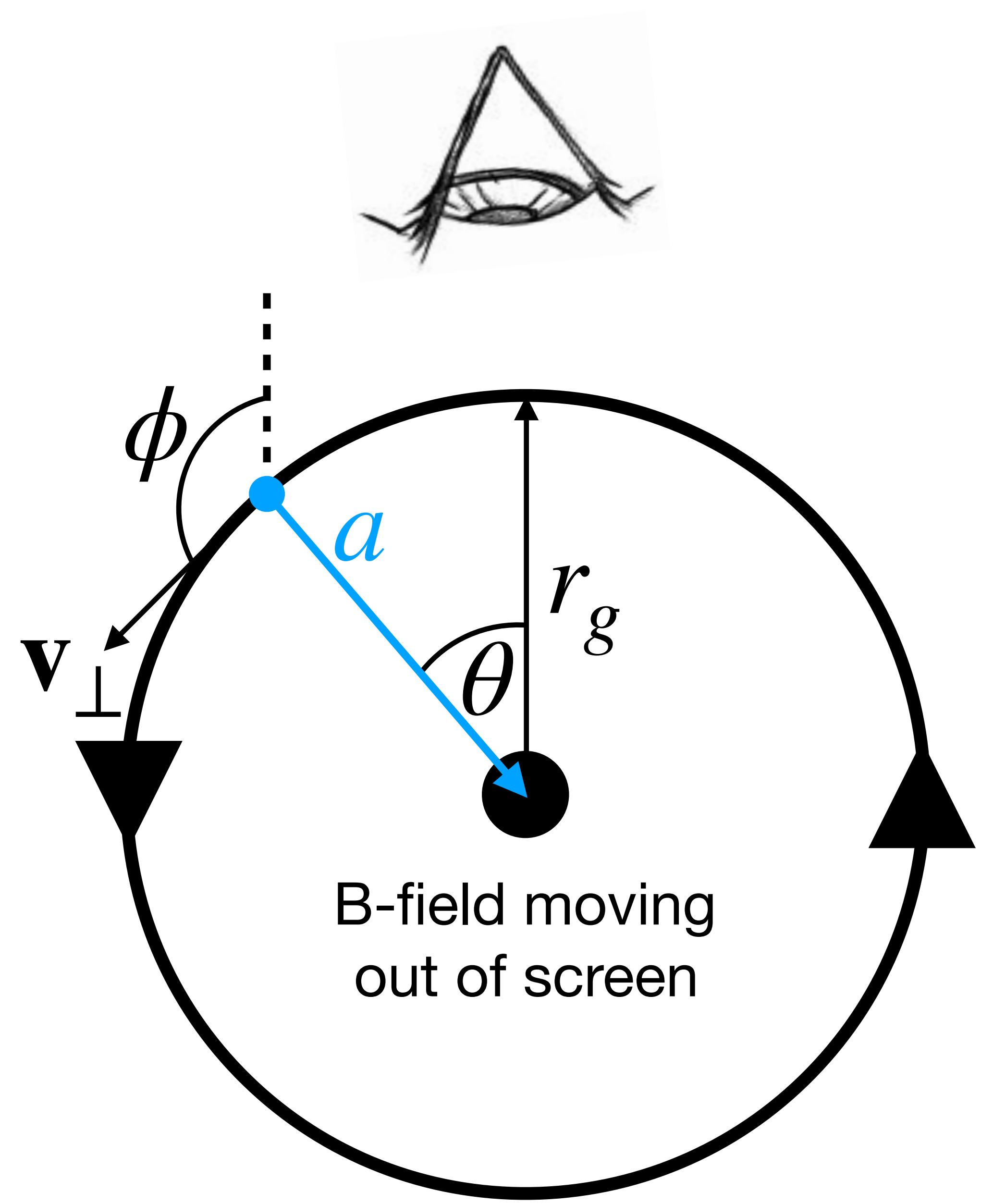

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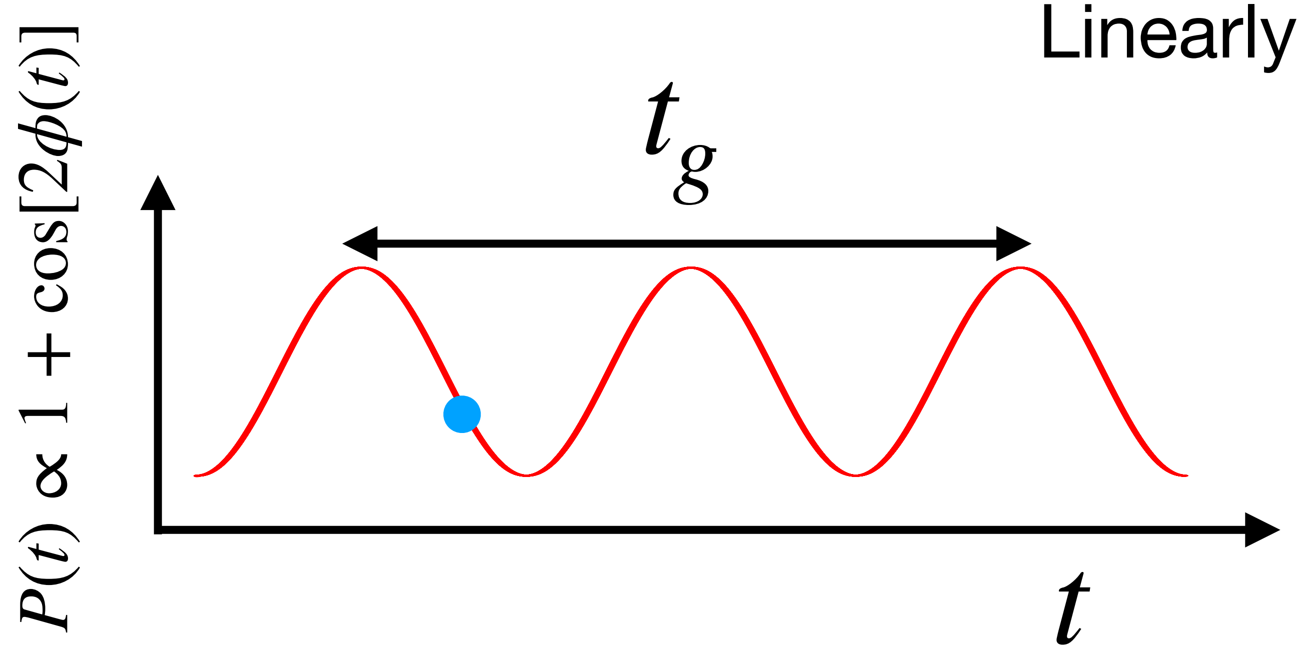
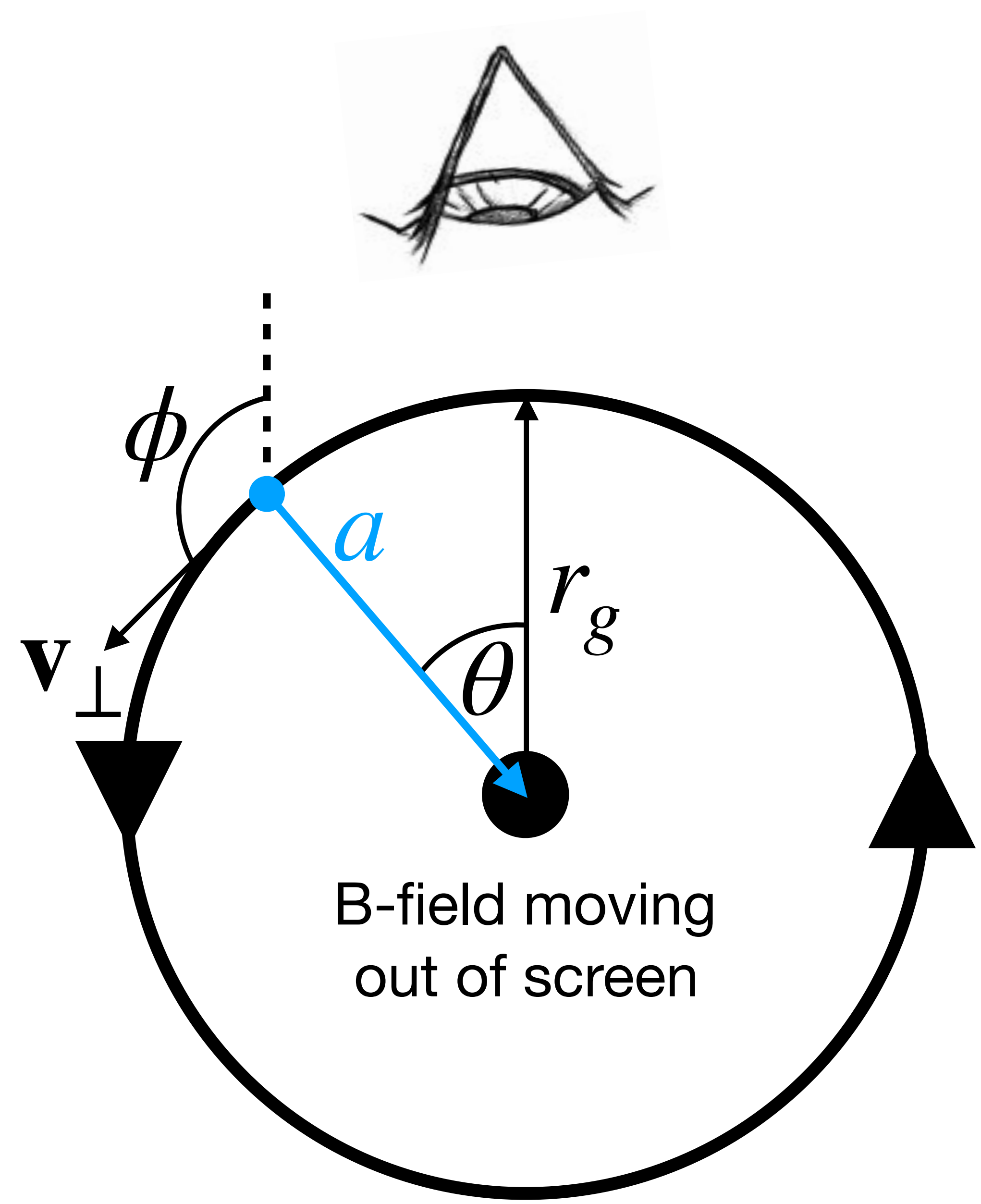
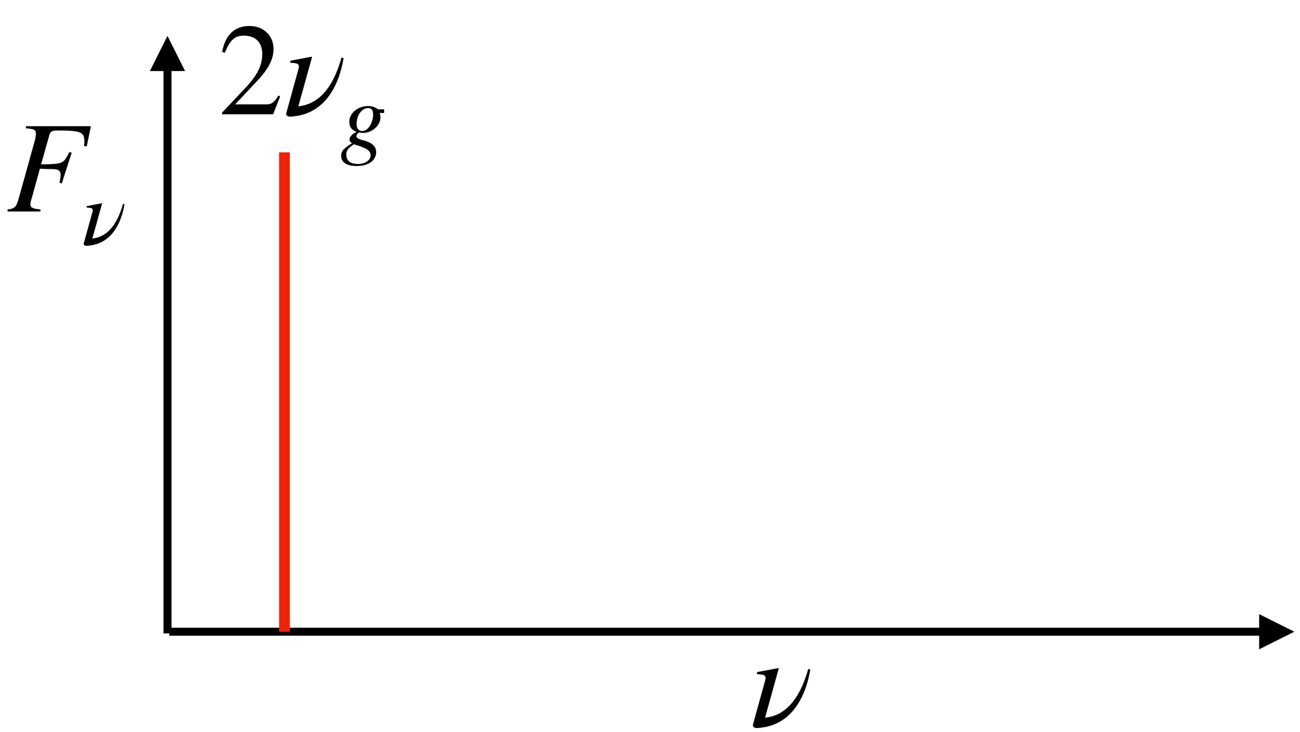


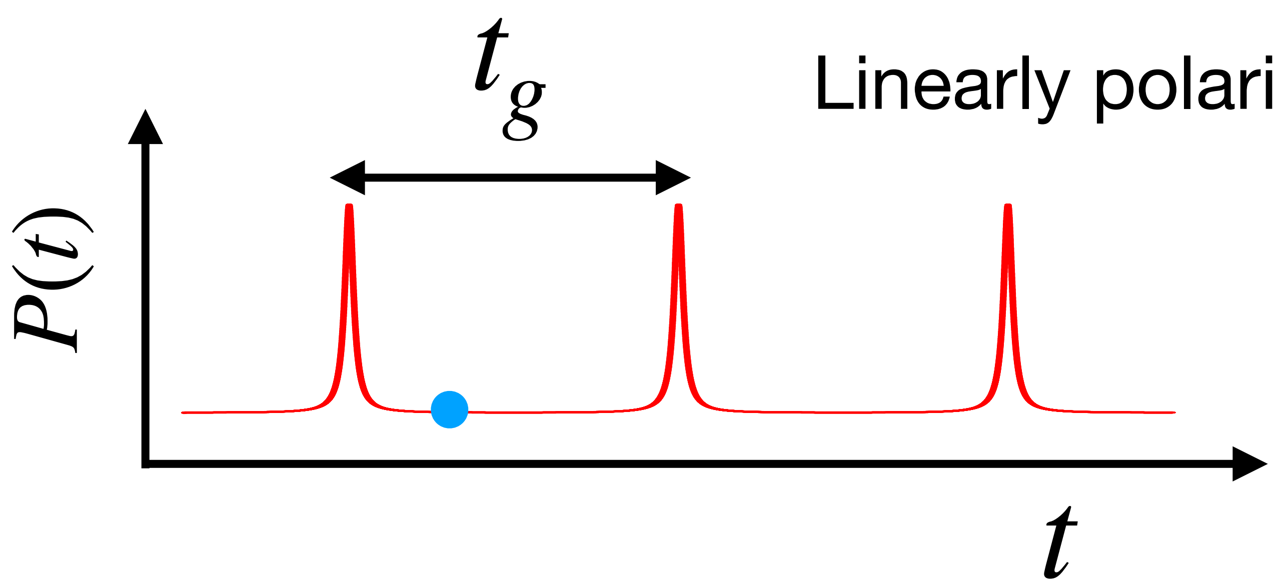


Fourier transform to get spectrum:

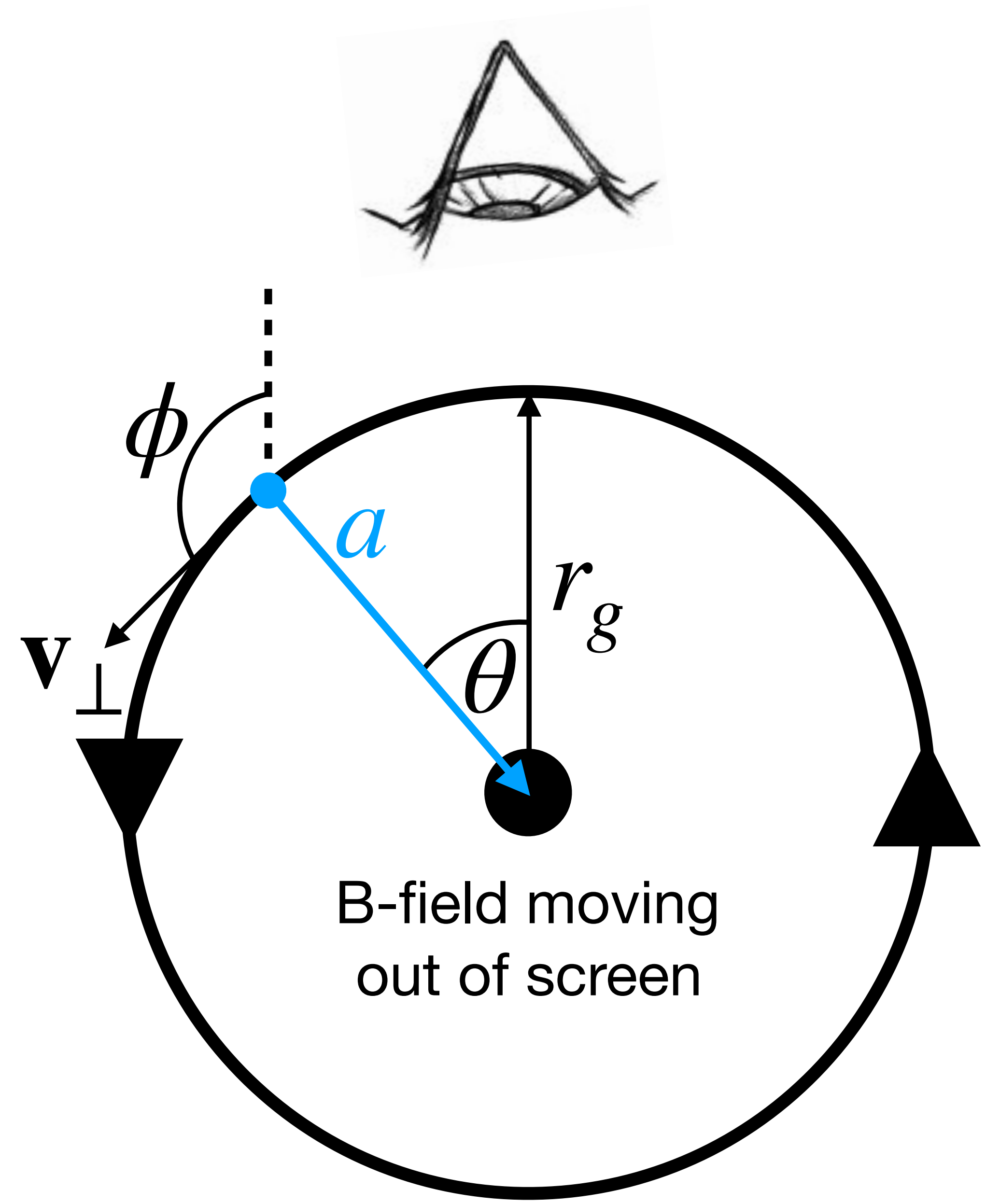


Spectrum

γ>>1: Relativistic beaming



Linearly polarised



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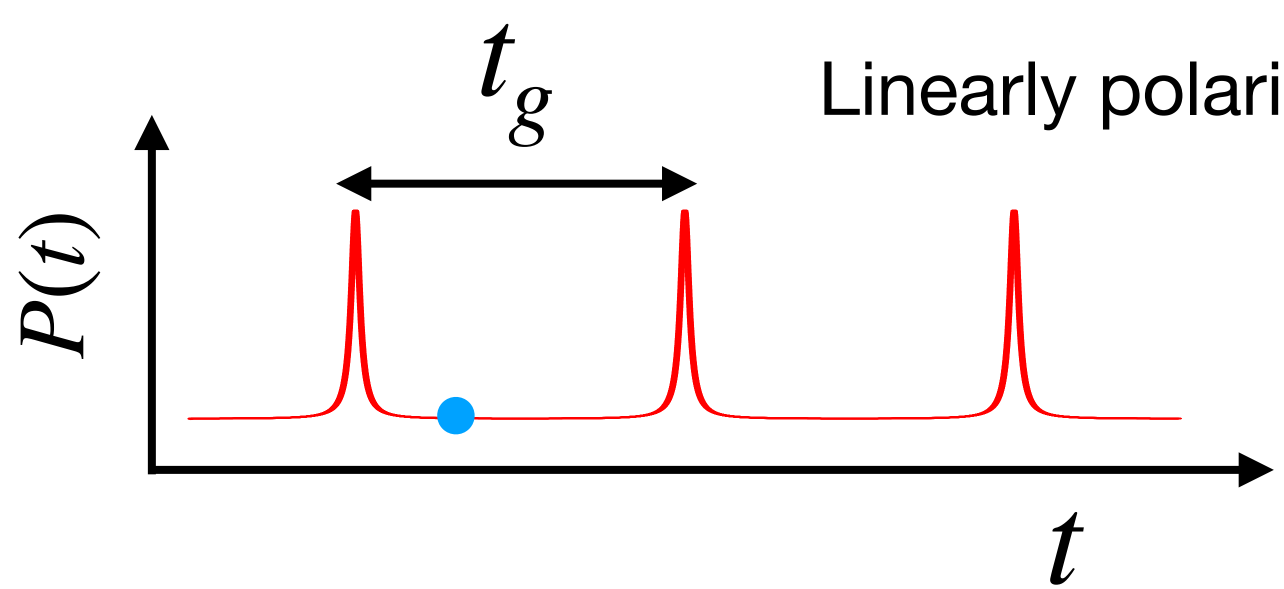
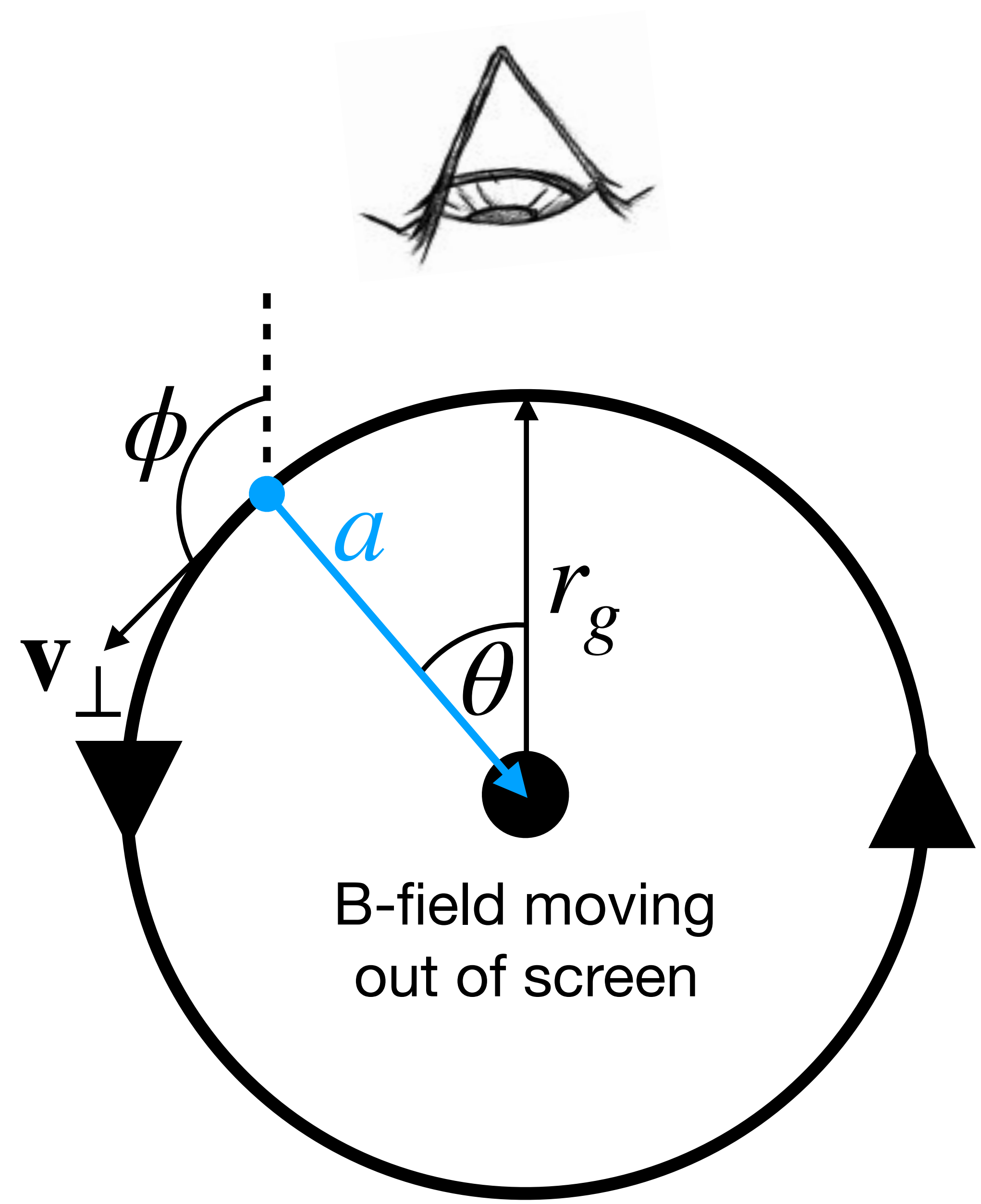


Linearly polarised



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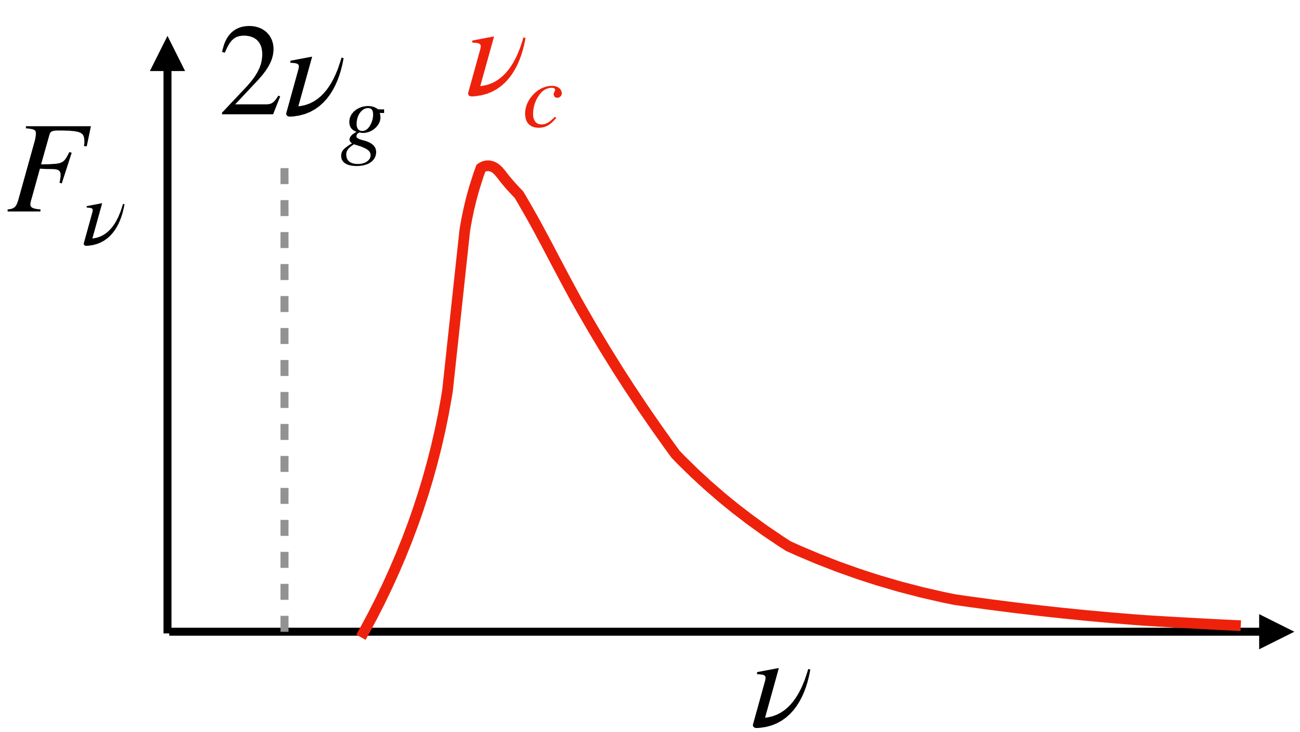

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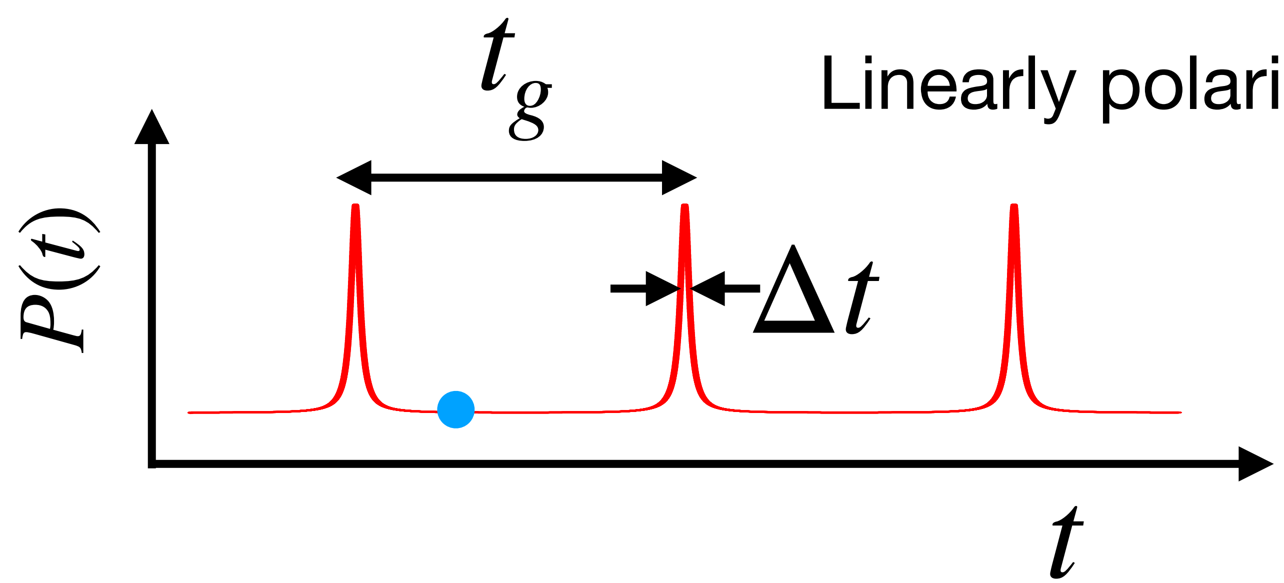

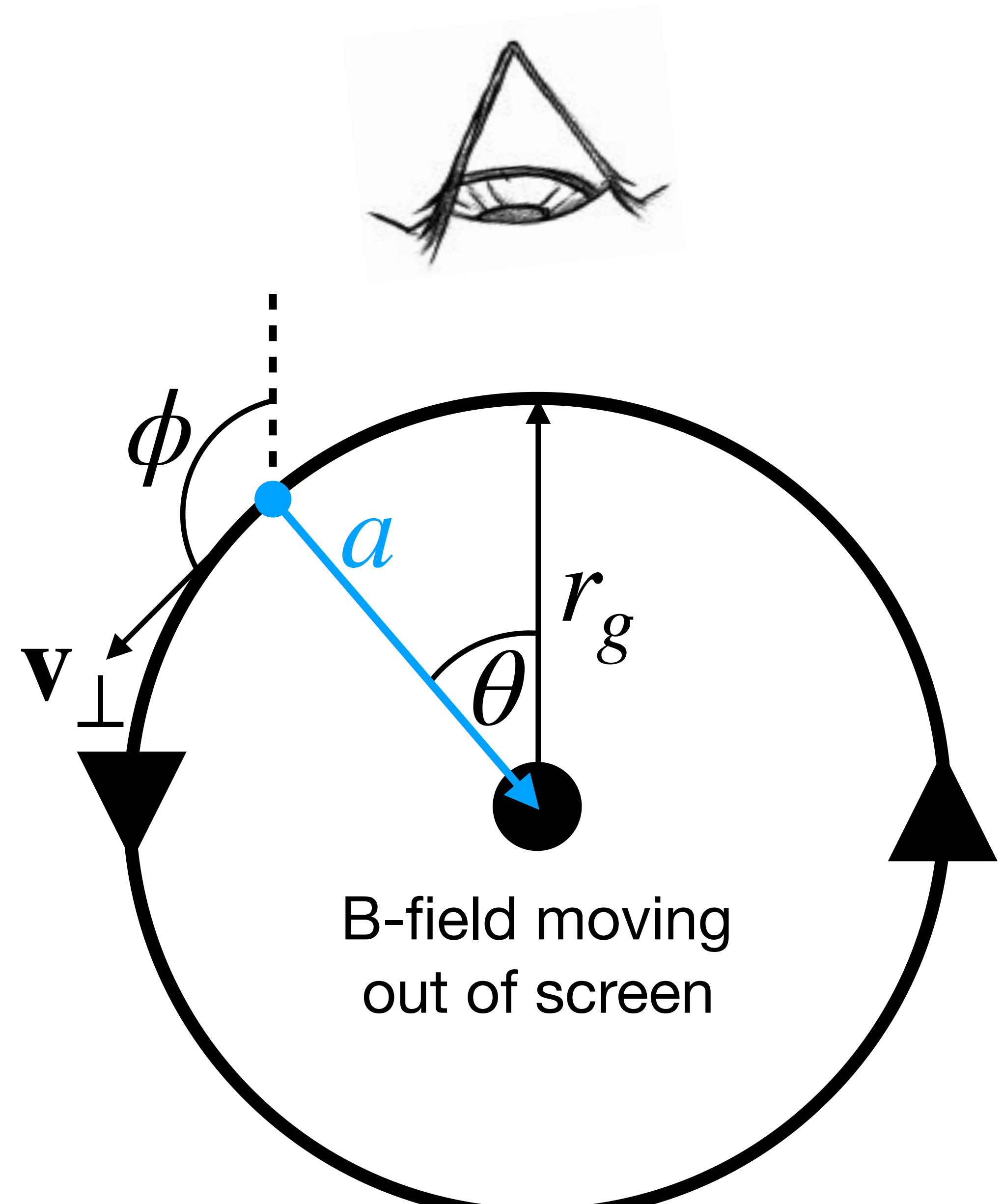

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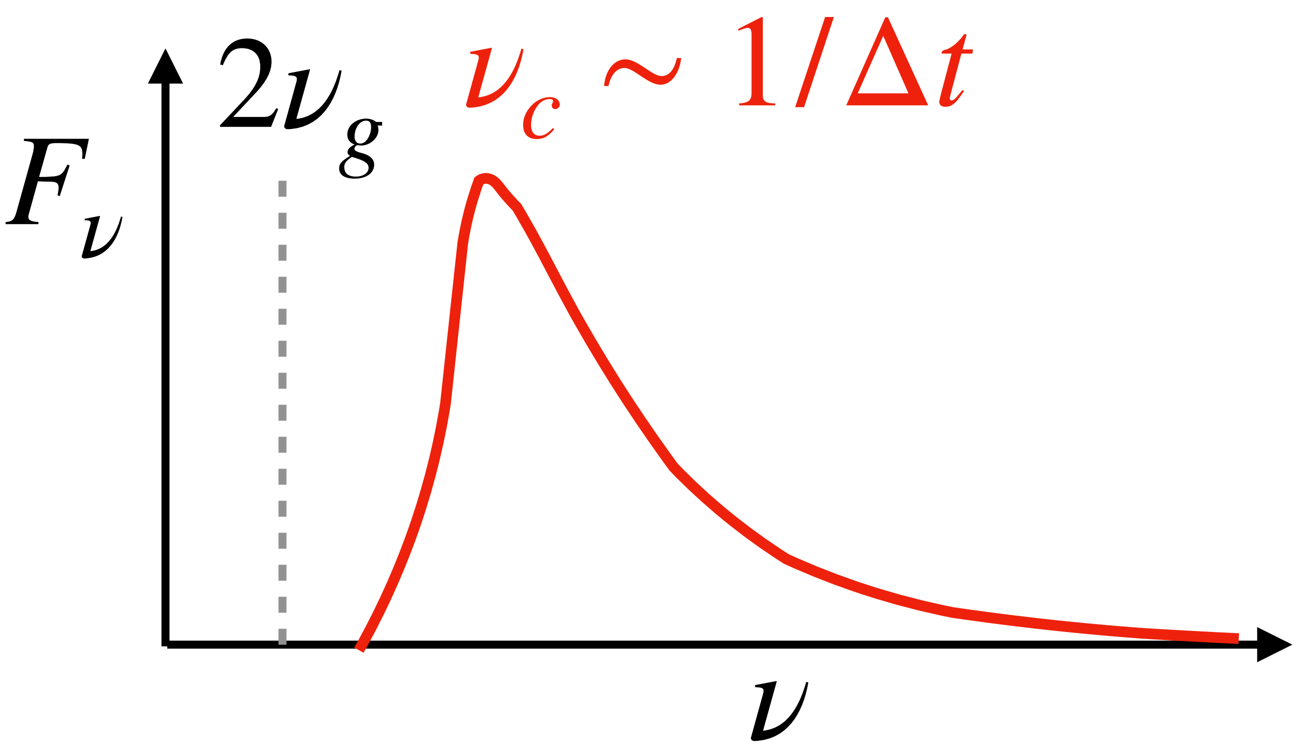


Spectrum

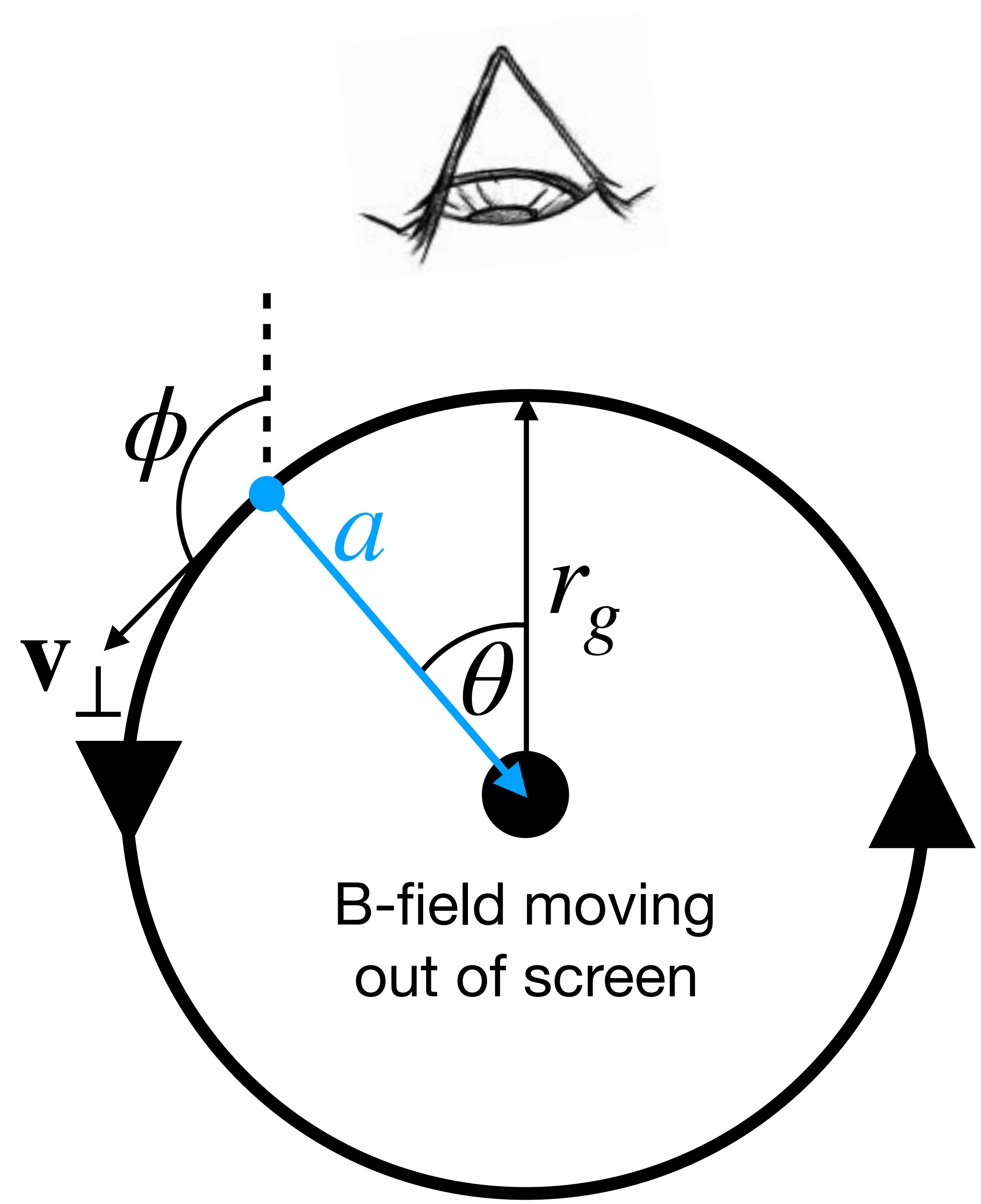
γ>>1: Relativistic beaming

Relativistic aberration formula:

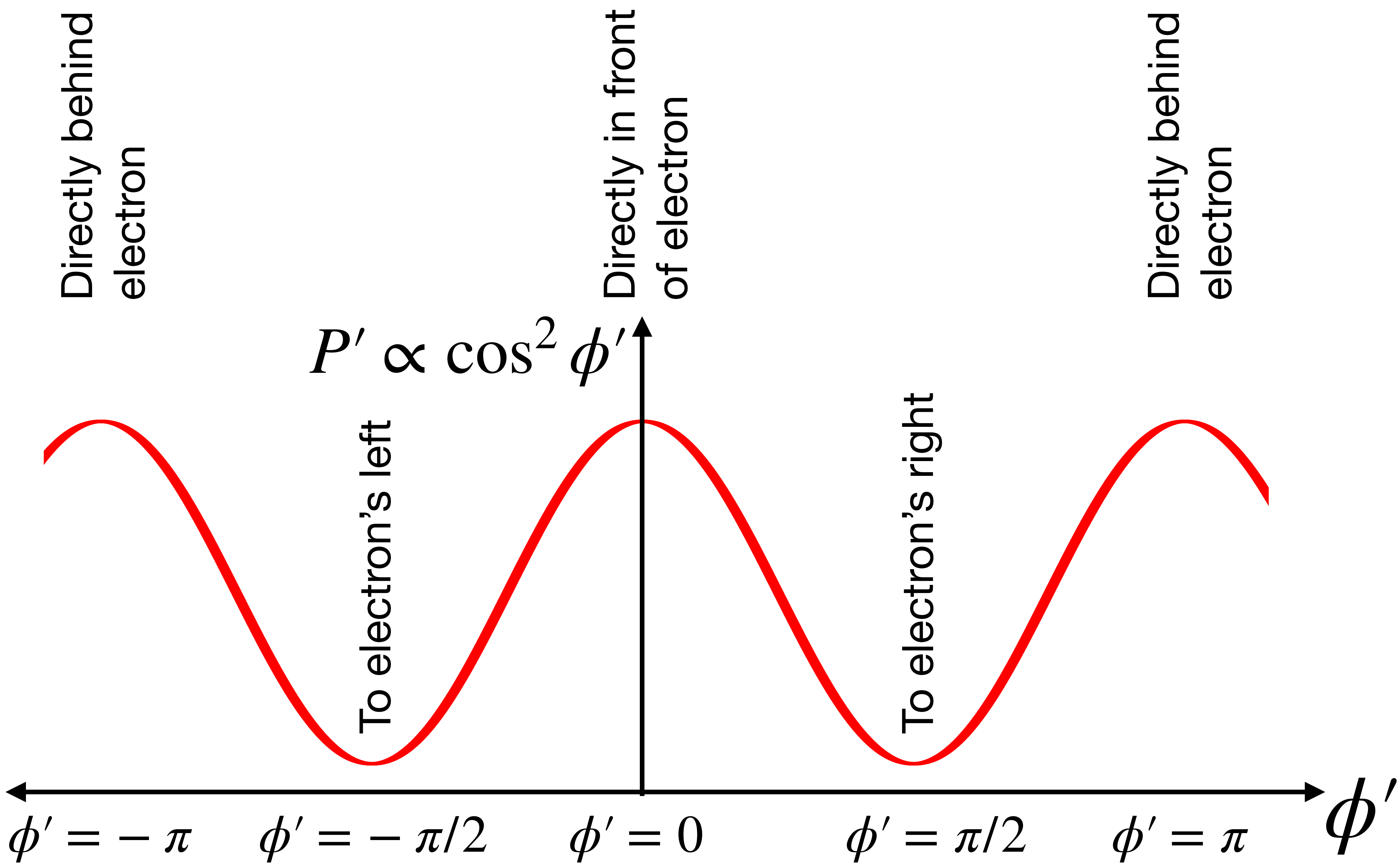
$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'}$$

Observer's rest frame: ϕ

Electron's rest frame: ϕ'

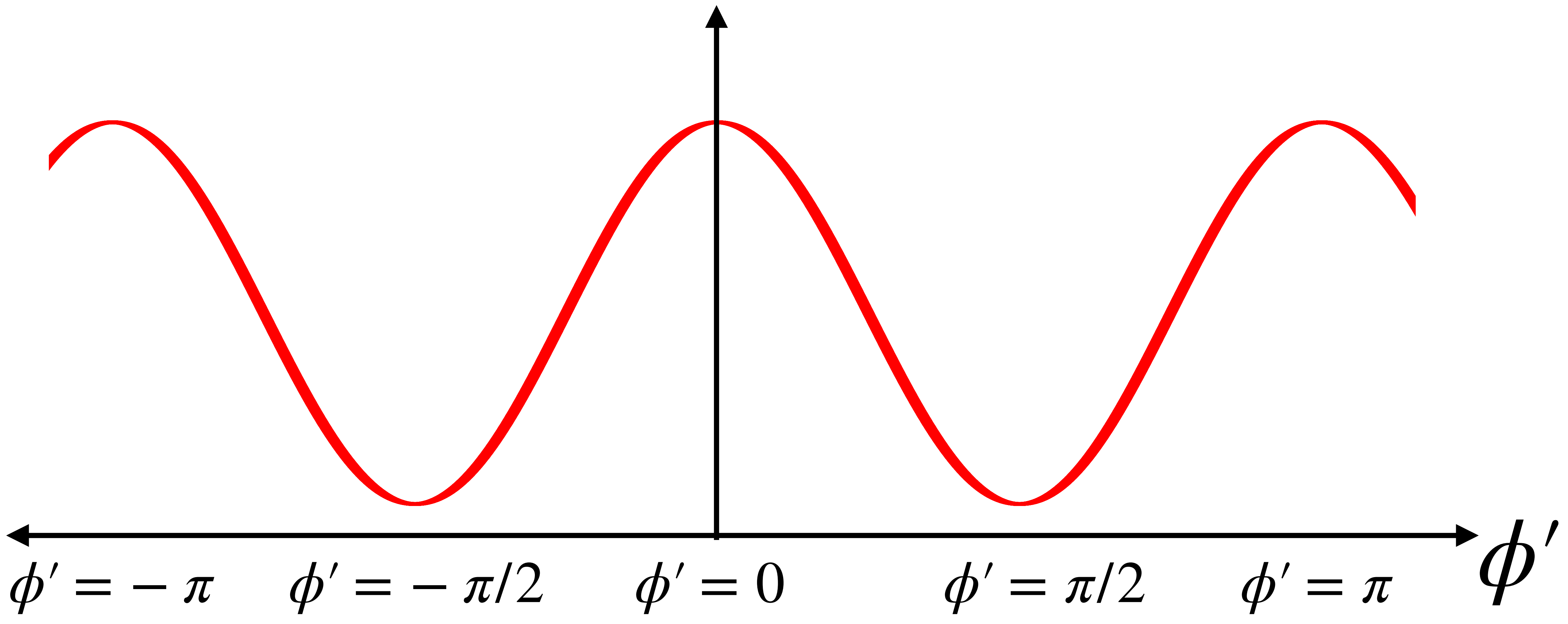


Relativistic aberration



Relativistic aberration

- P(t) peaks at $\phi' = 0$ and is zero at $\phi' = \pi/2$ and $\phi' = -\pi/2$



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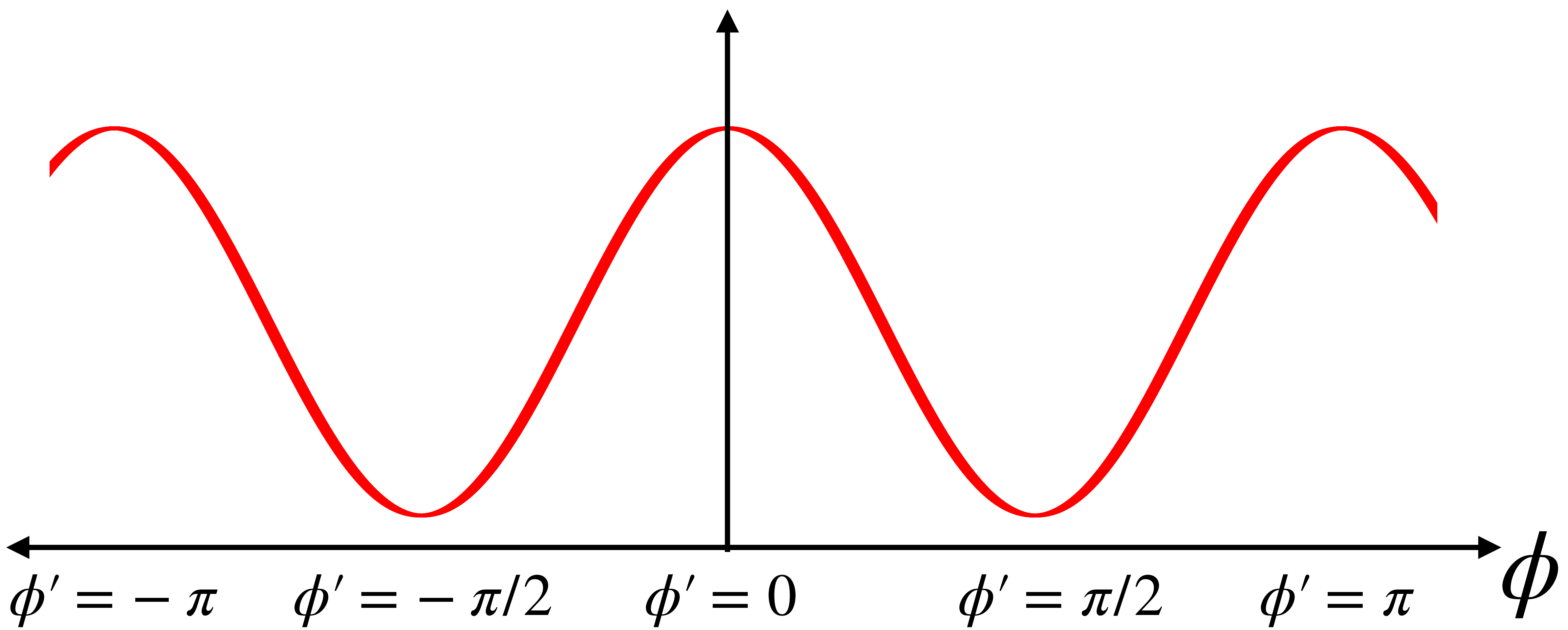

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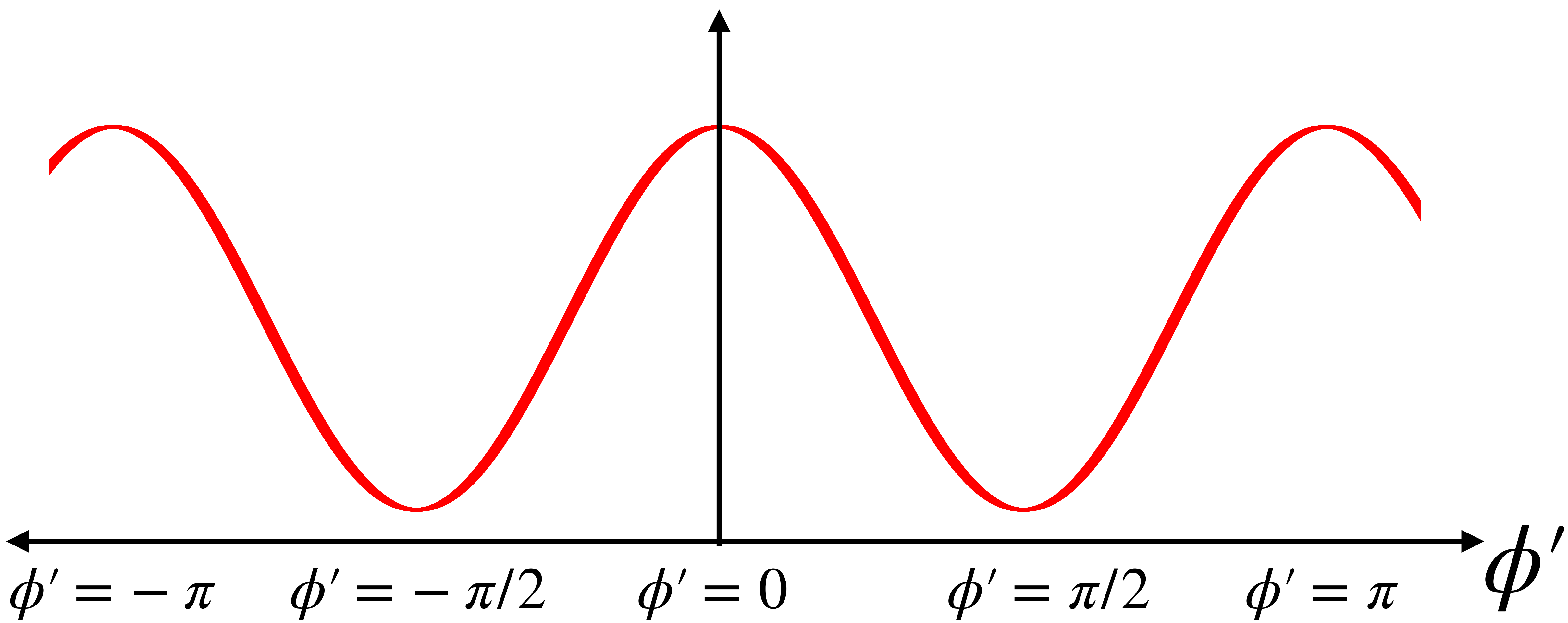

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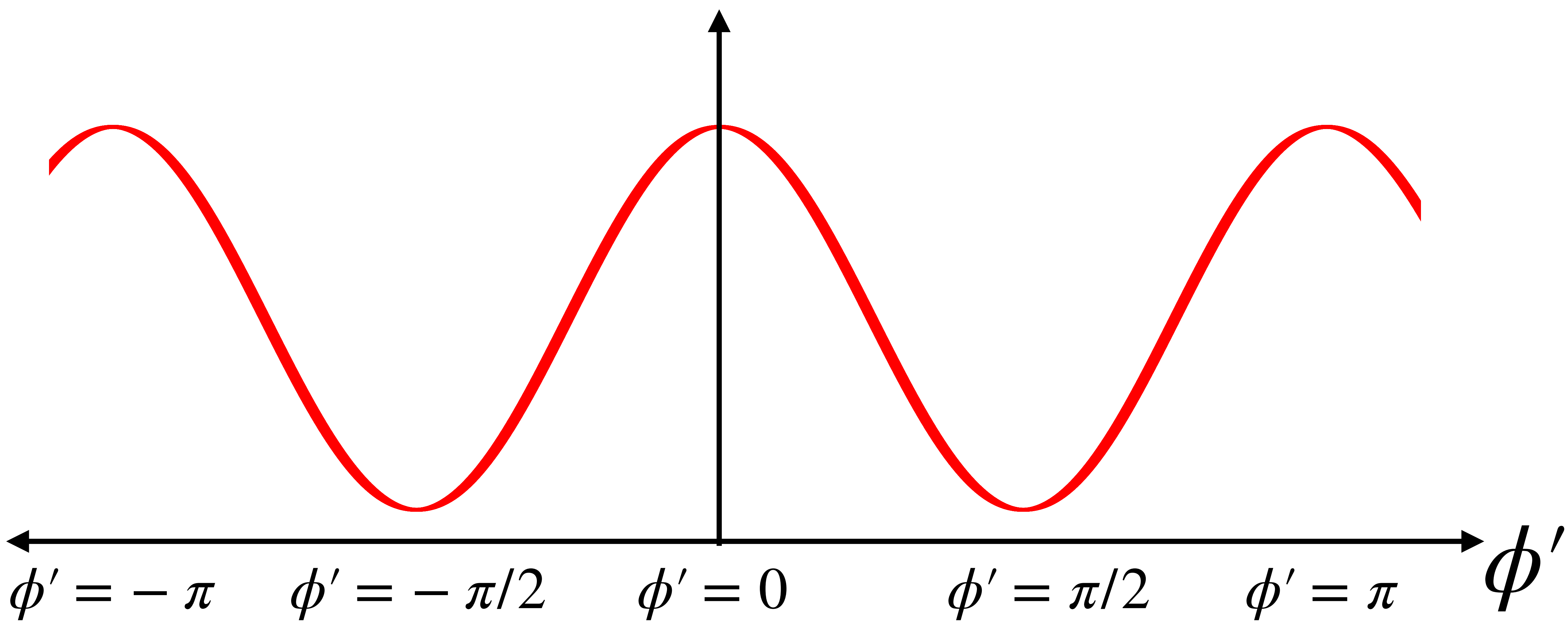

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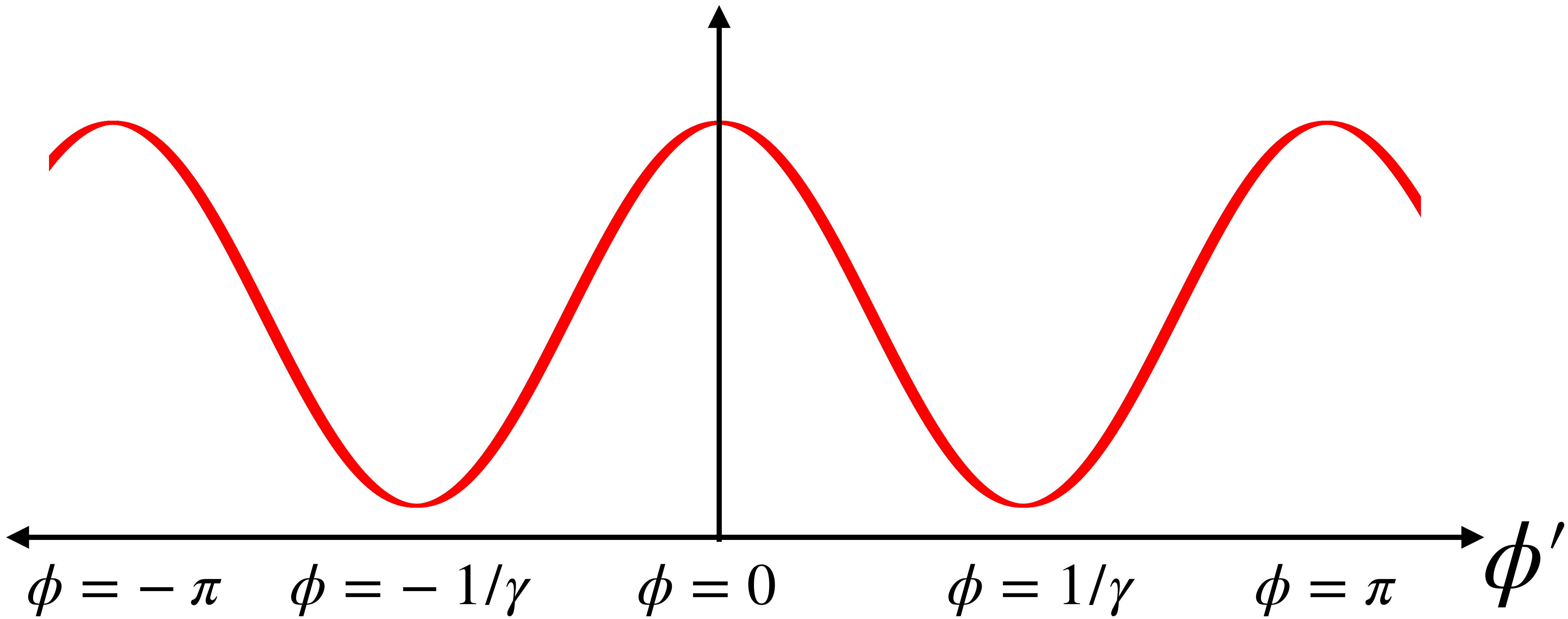

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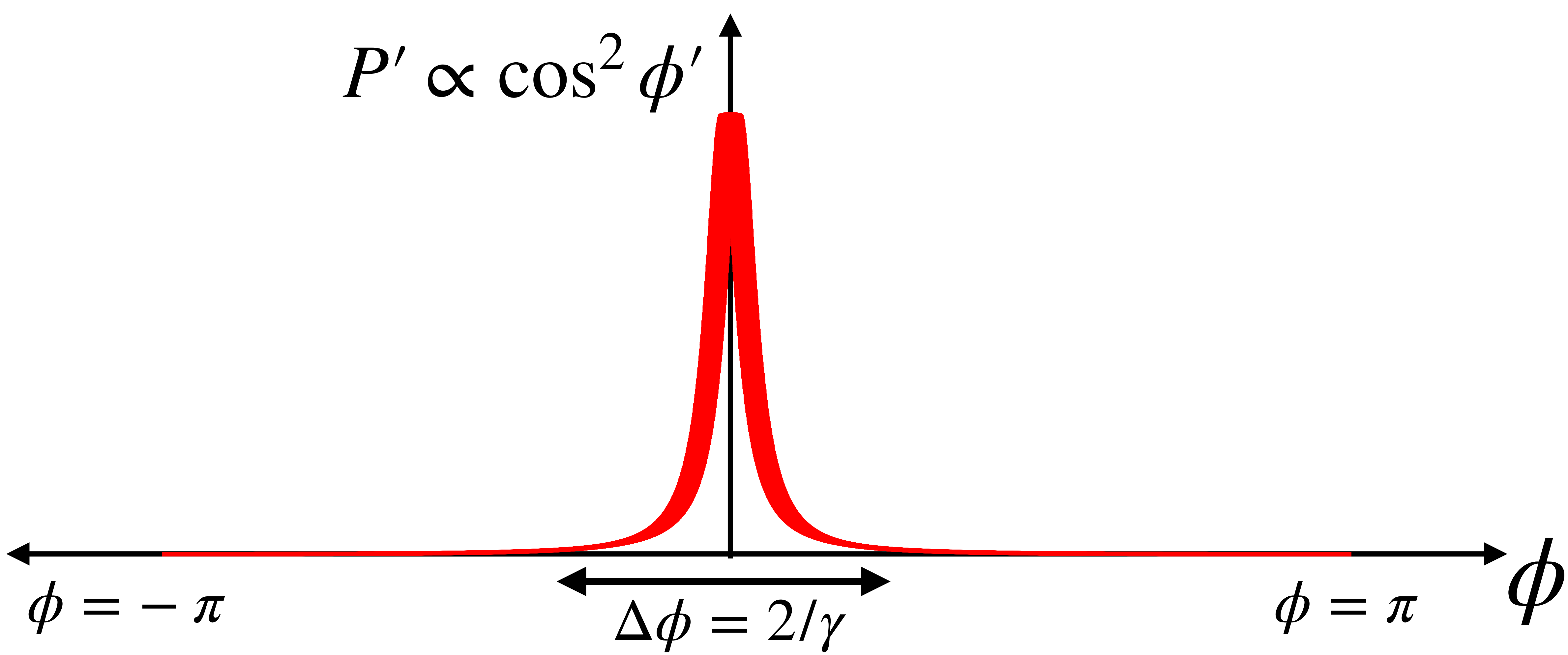

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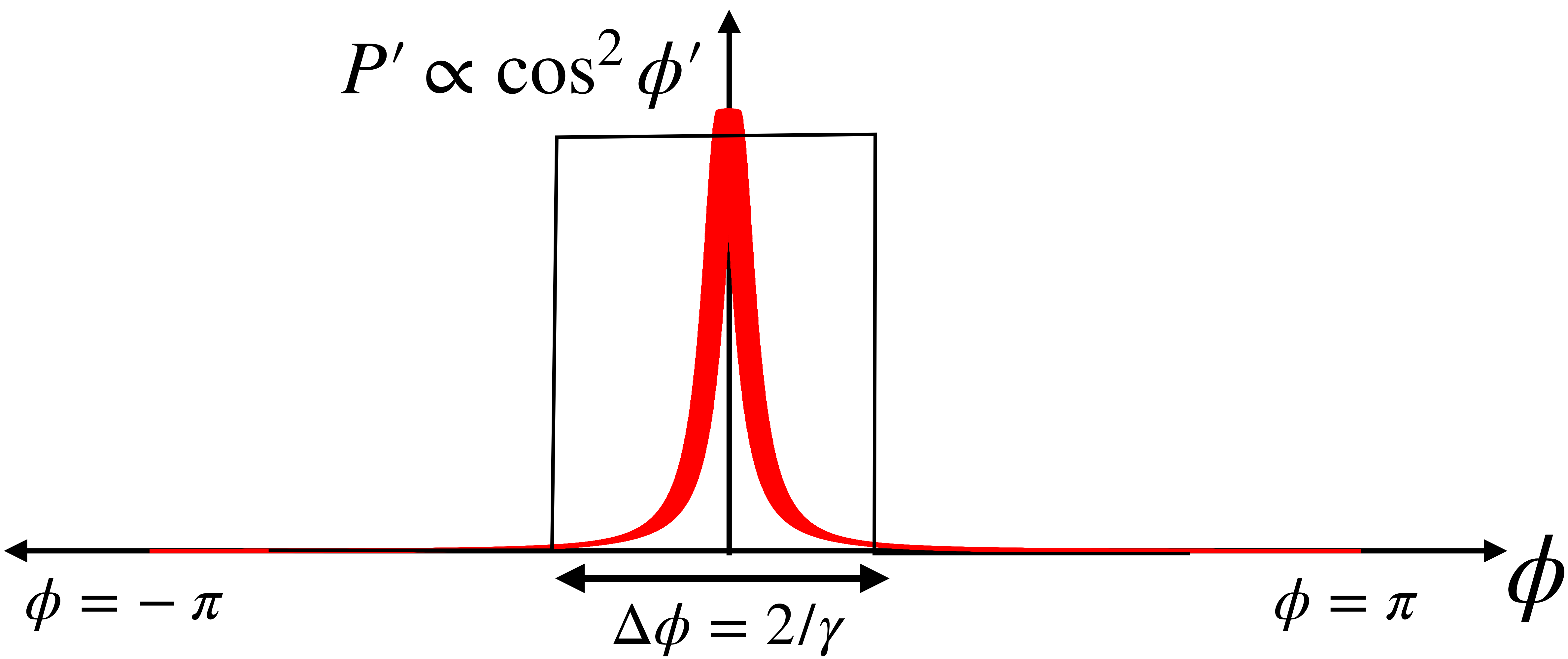
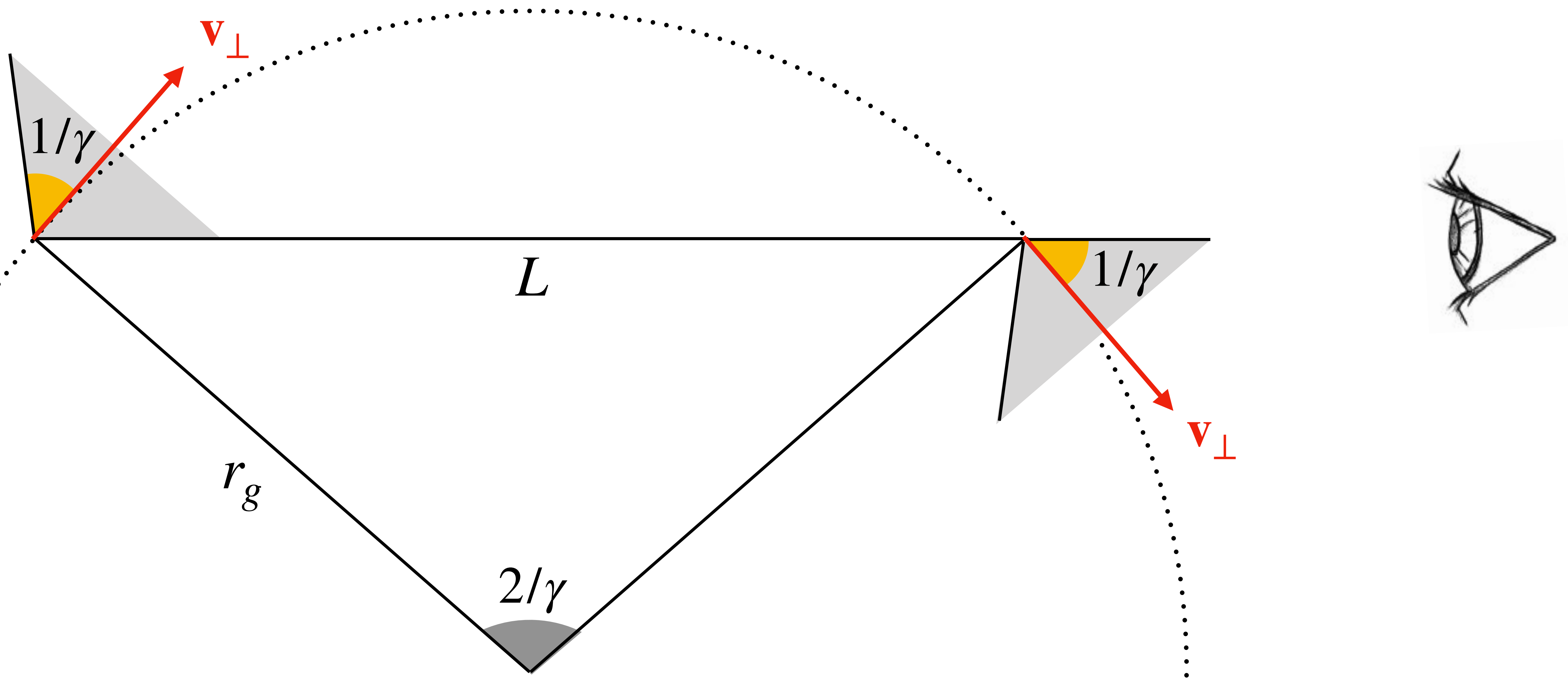


Relativistic aberration

- Electron orbits an angle 2/γ from our sightline first cutting the cone to our sightline first not cutting the cone.
- Electron travels this distance in: $\delta t = (2/\gamma)/\omega_g$



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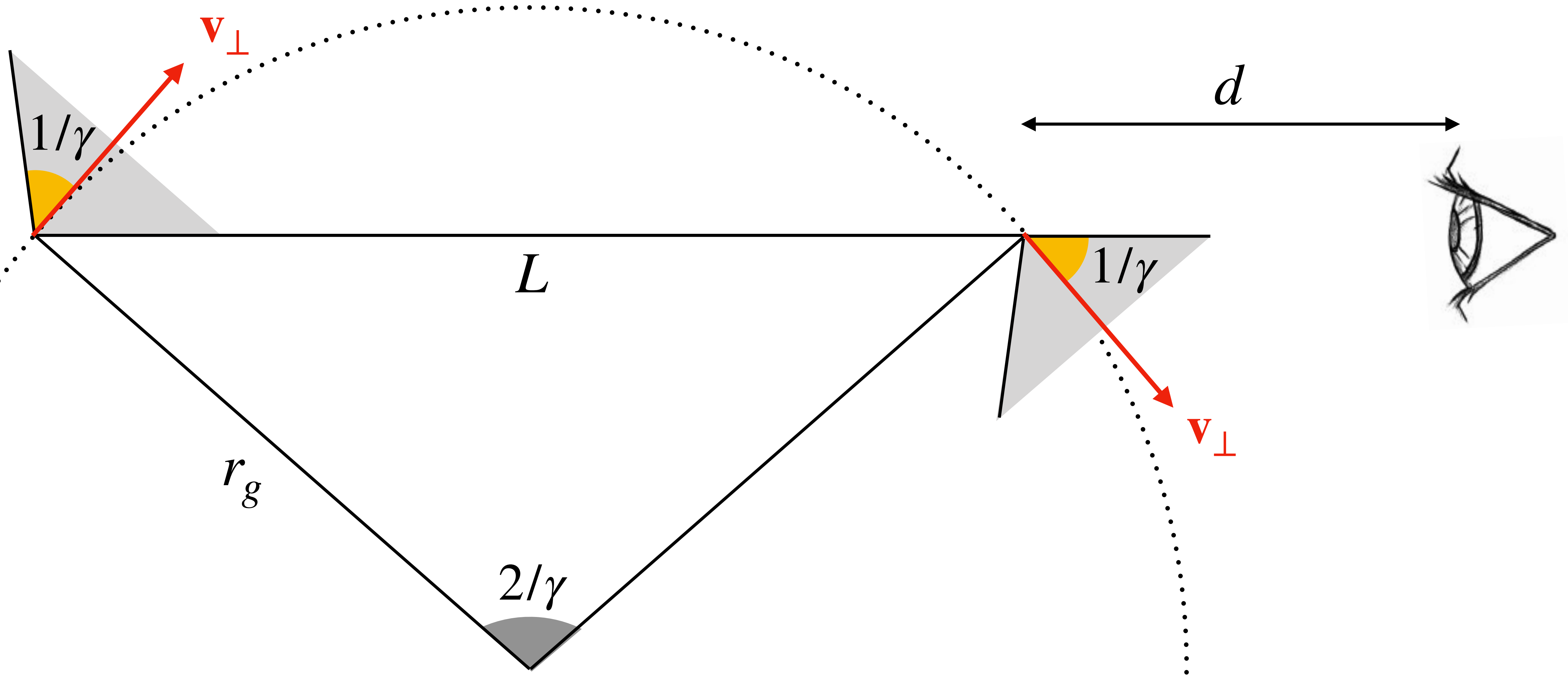

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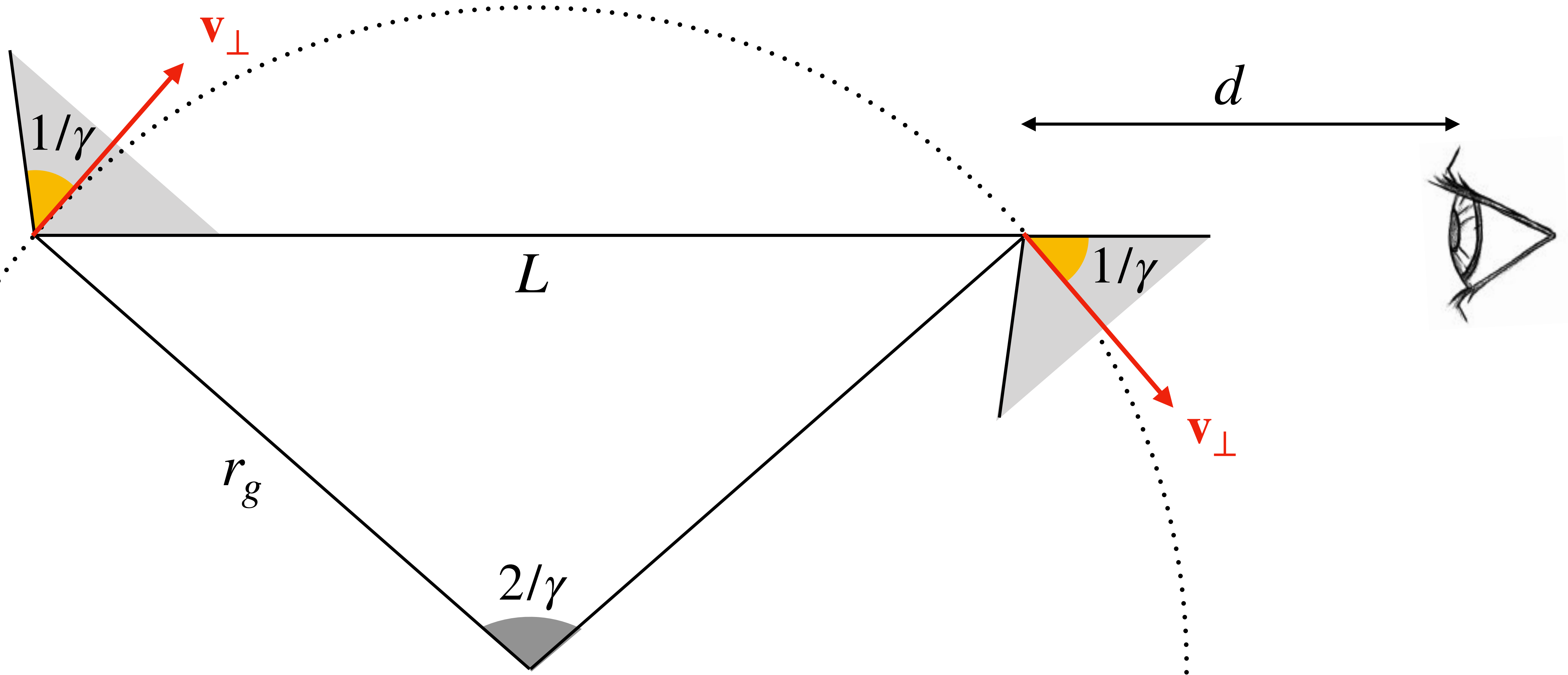

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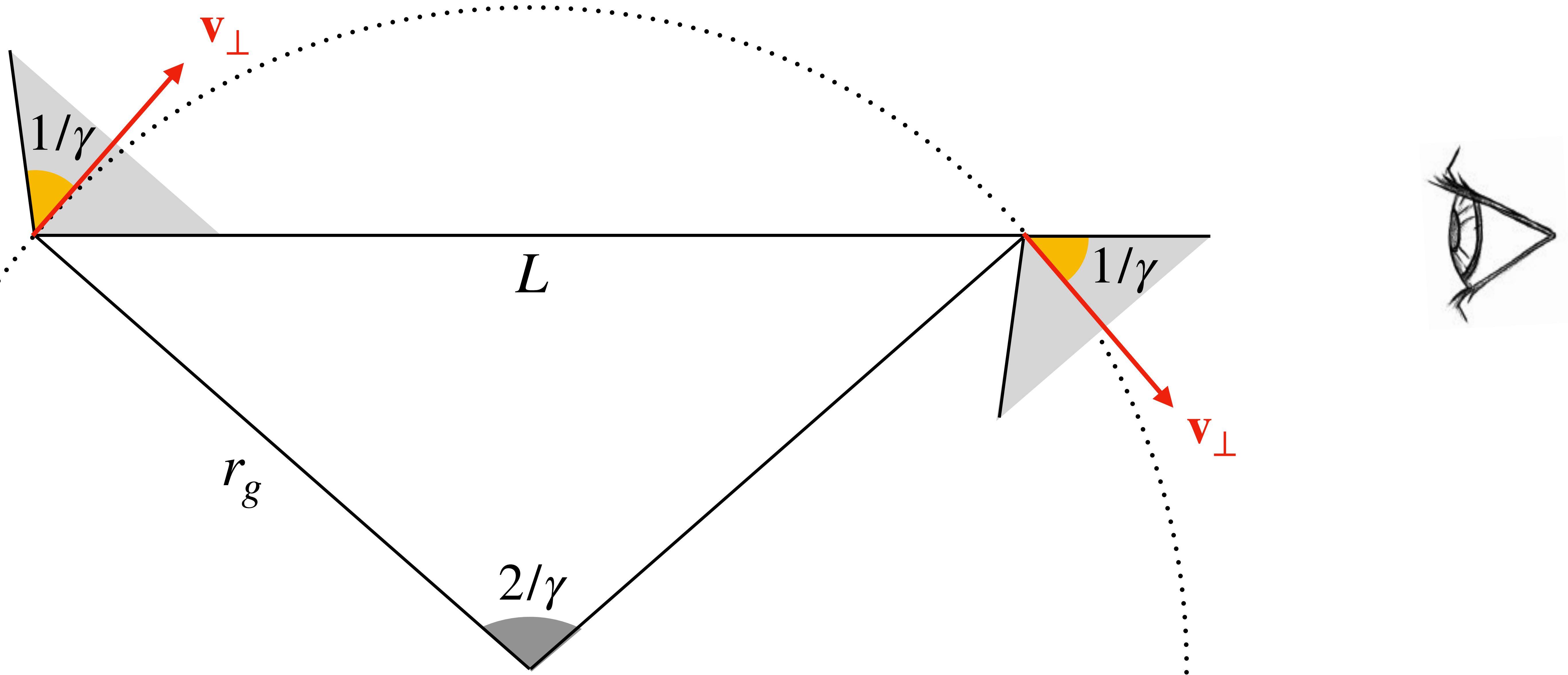
- Therefore length of pulse is: $\Delta t = t_2 - t_1 = \delta t - L/c$



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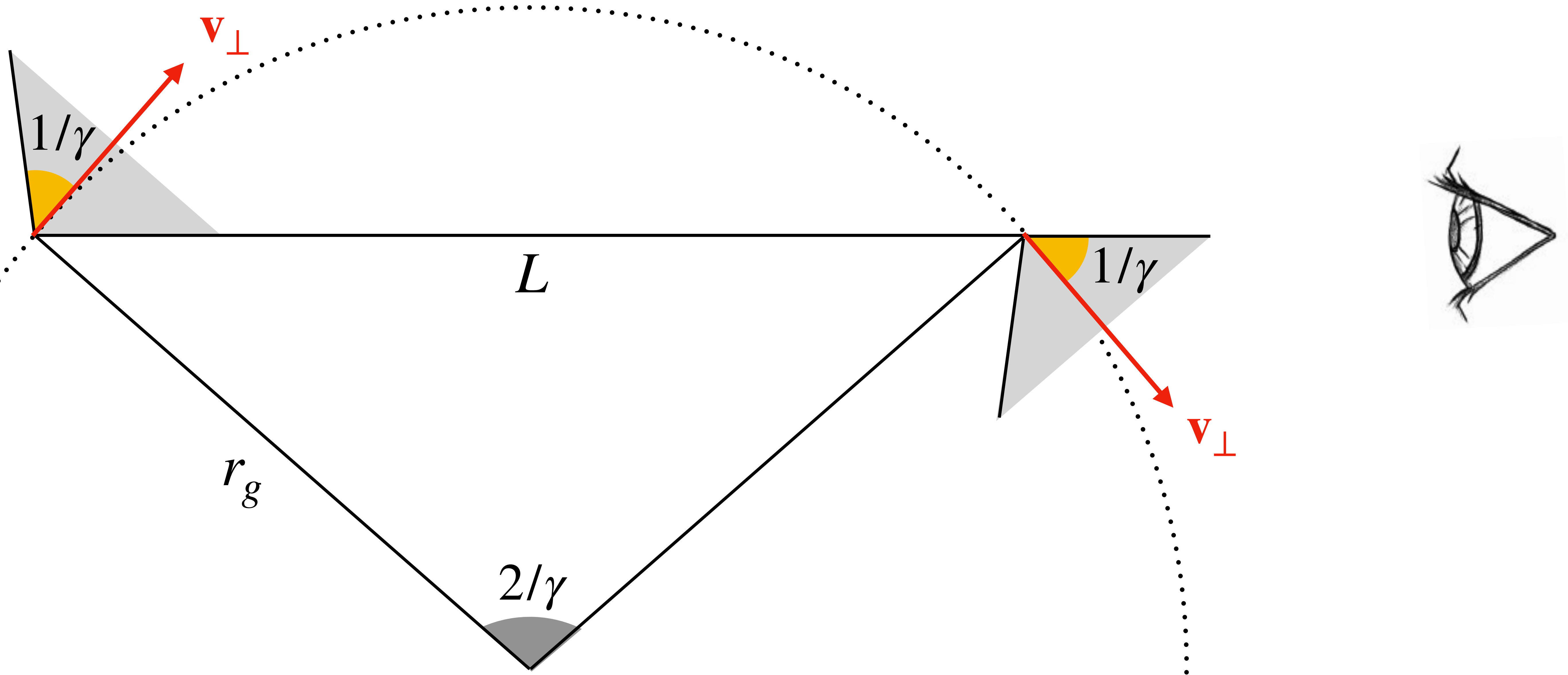

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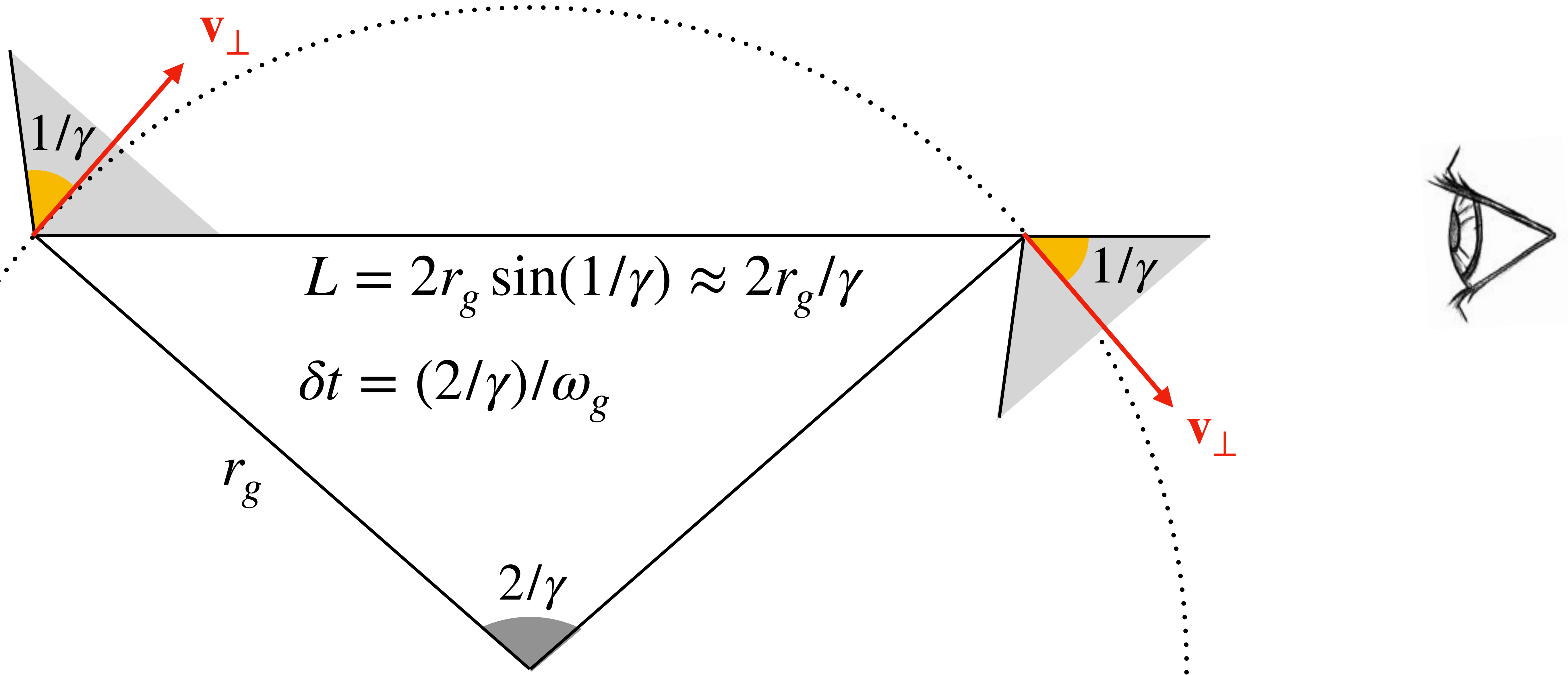

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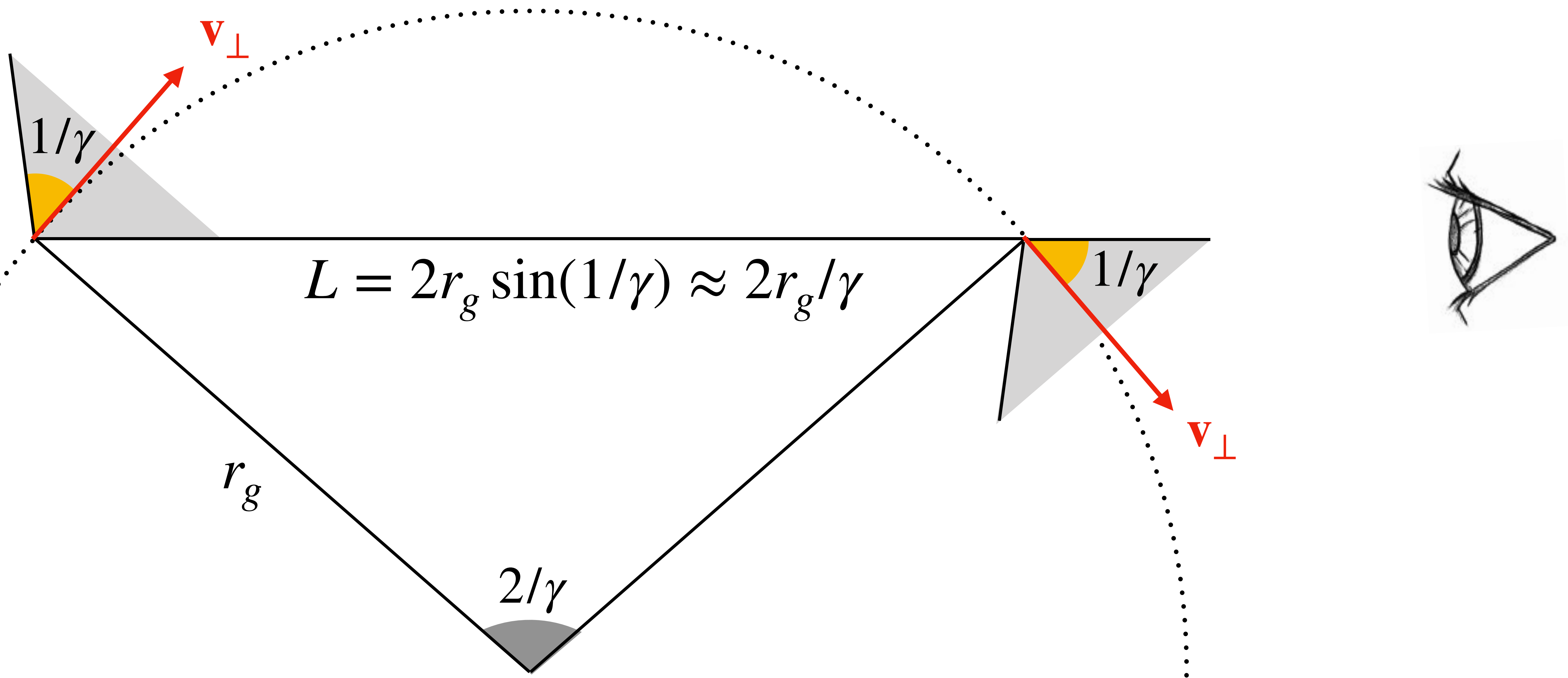

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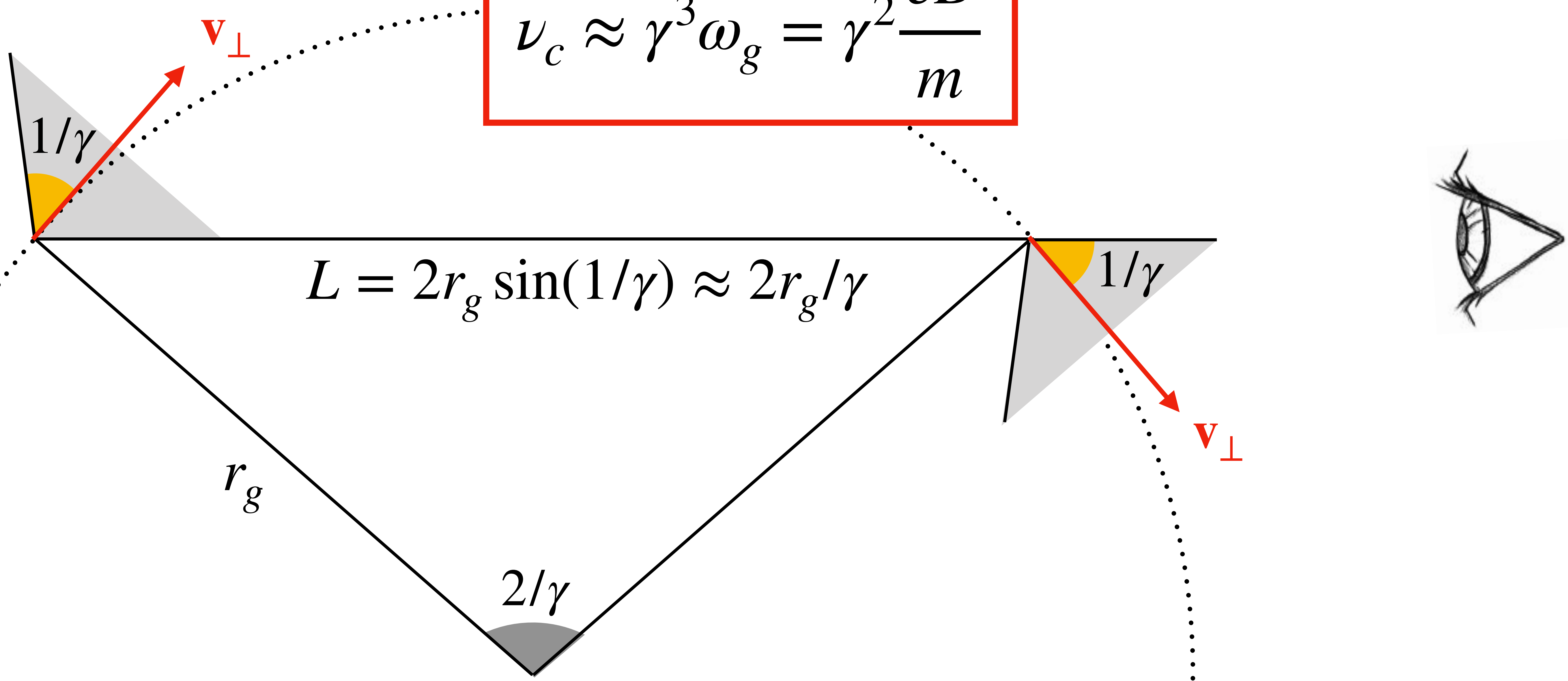
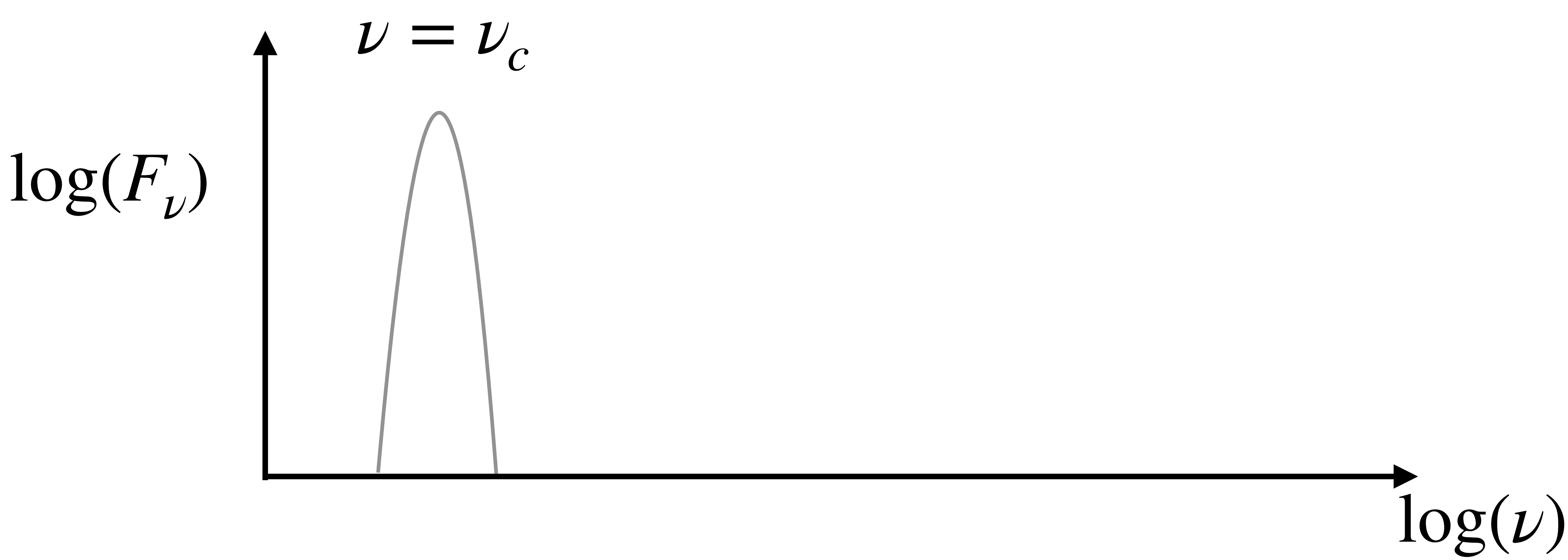
$$\nu_c \approx \gamma^3 \omega_g = \gamma^2 \frac{eB}{m}$$



Population of electrons

- Spectrum for electrons with energy $E \propto \gamma$ is narrow and peaked at $\nu \propto \gamma^2$; peak value is $P \propto \gamma^2$



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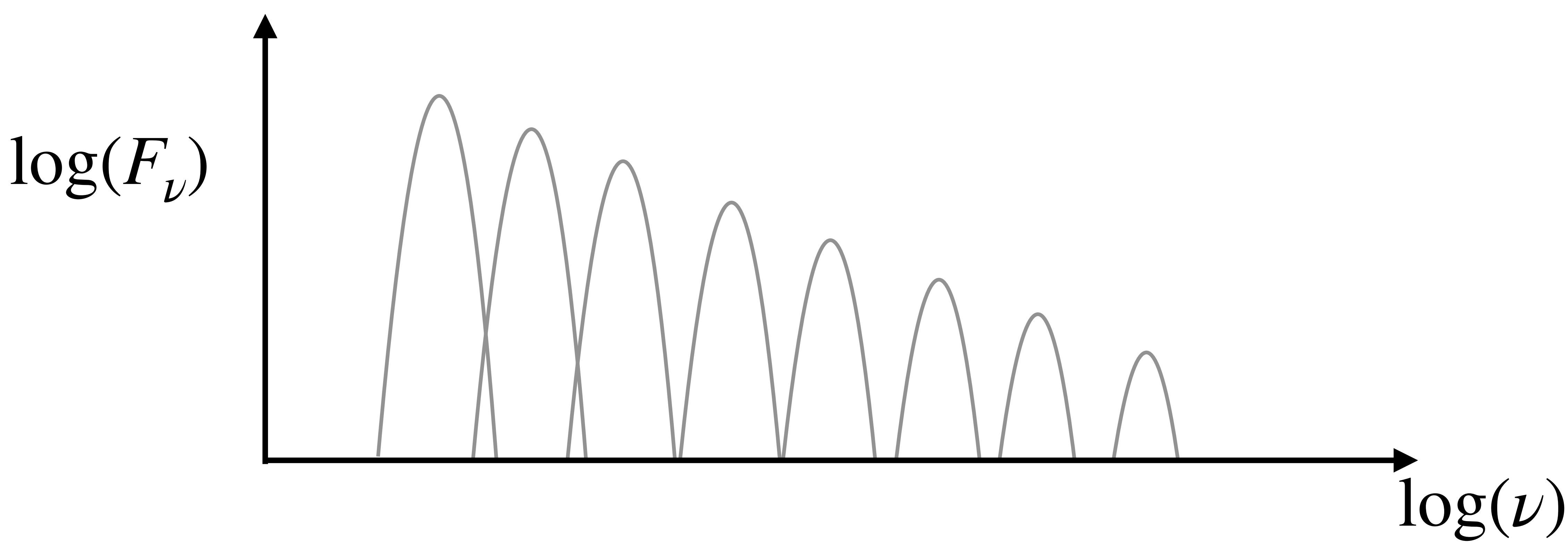

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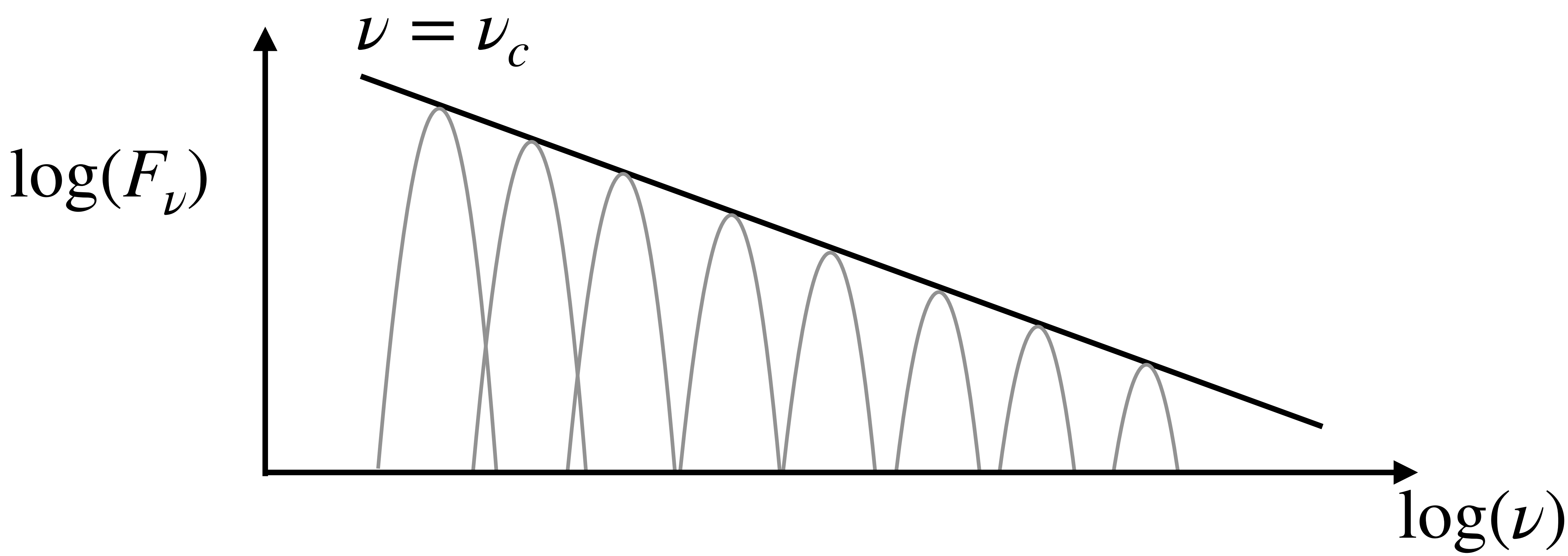

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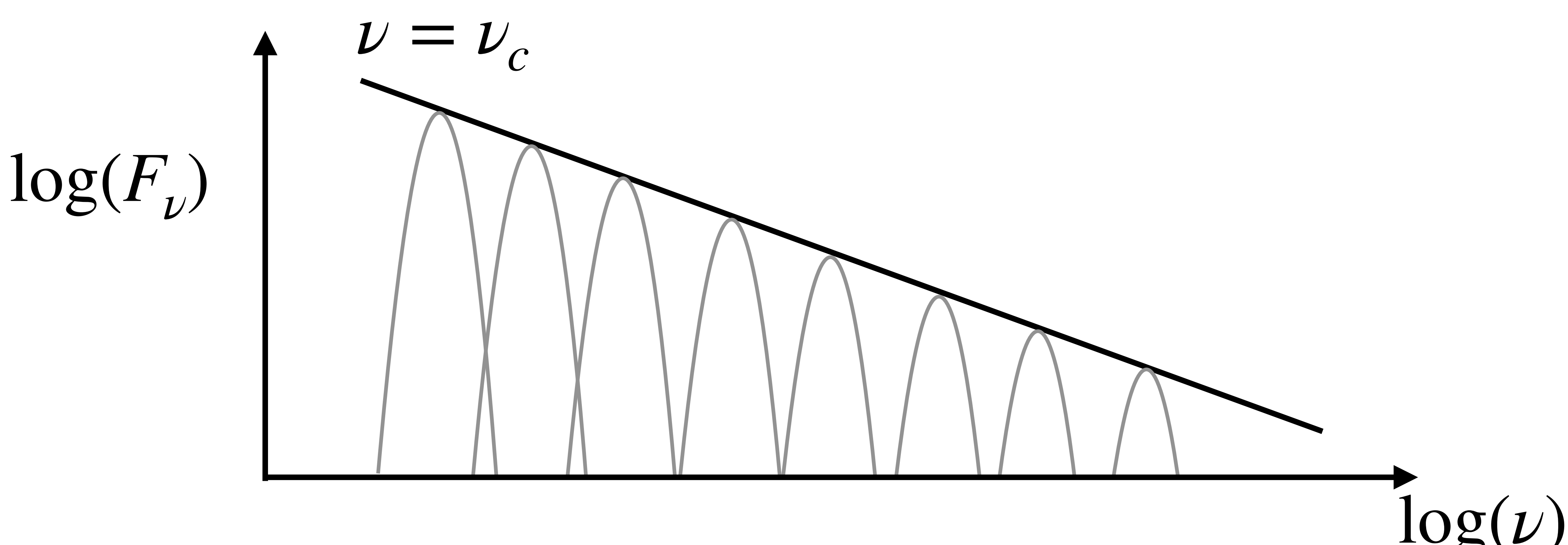

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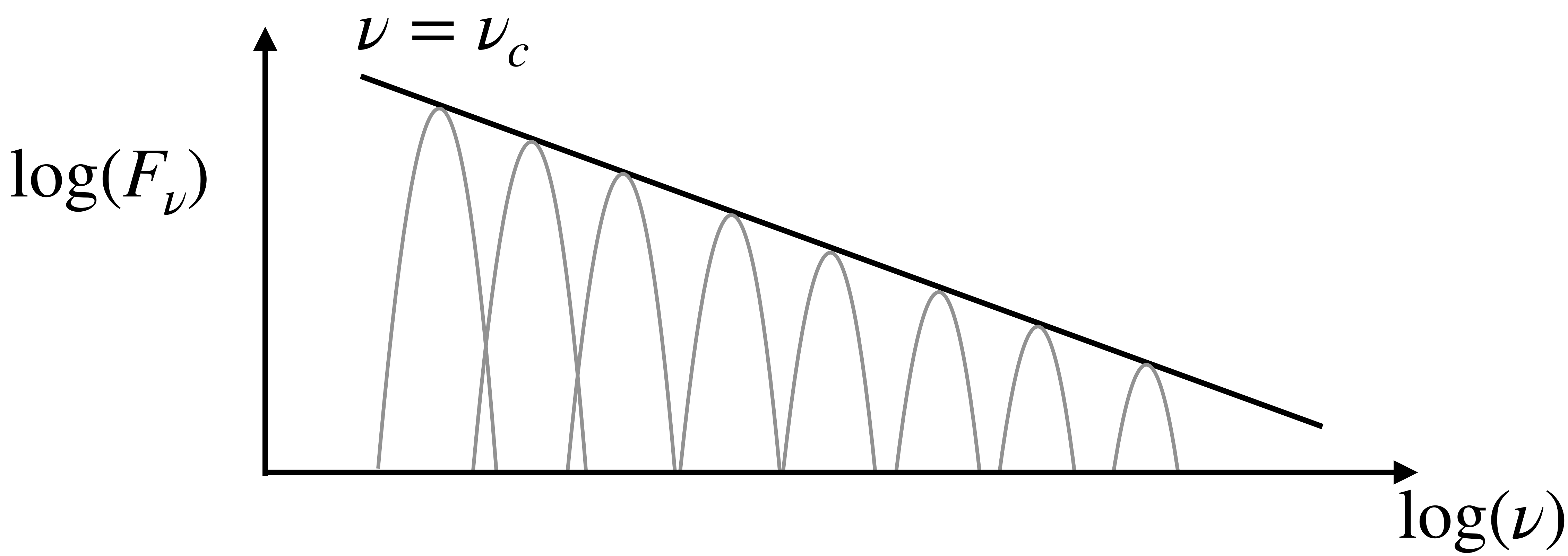
Hint: $F_\nu = \dfrac{dF}{d\nu} \propto P\dfrac{dN}{d\nu} \propto P\dfrac{dN}{d\gamma}\dfrac{d\gamma}{d\nu}$



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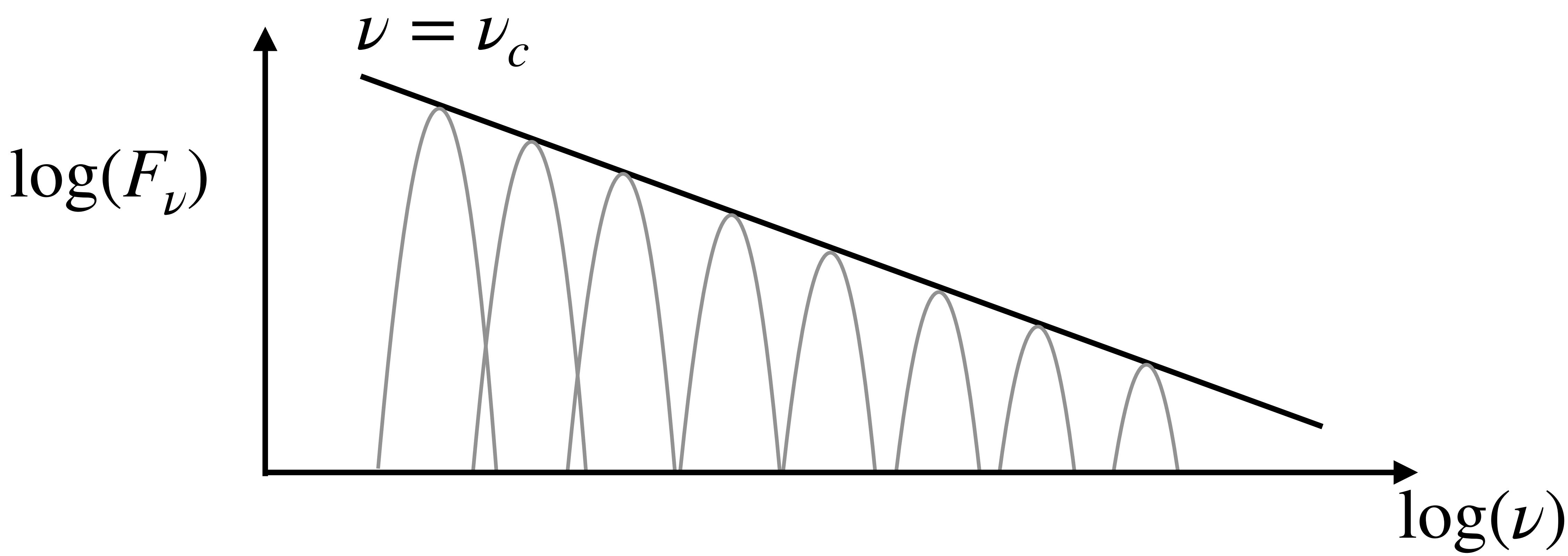

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- $k = 2 \implies \alpha = 0.5$ …we sometimes measure this for radio sources!
- But often measure $\alpha > 0.5 \implies k > 2$

…same as cosmic ray spectrum!