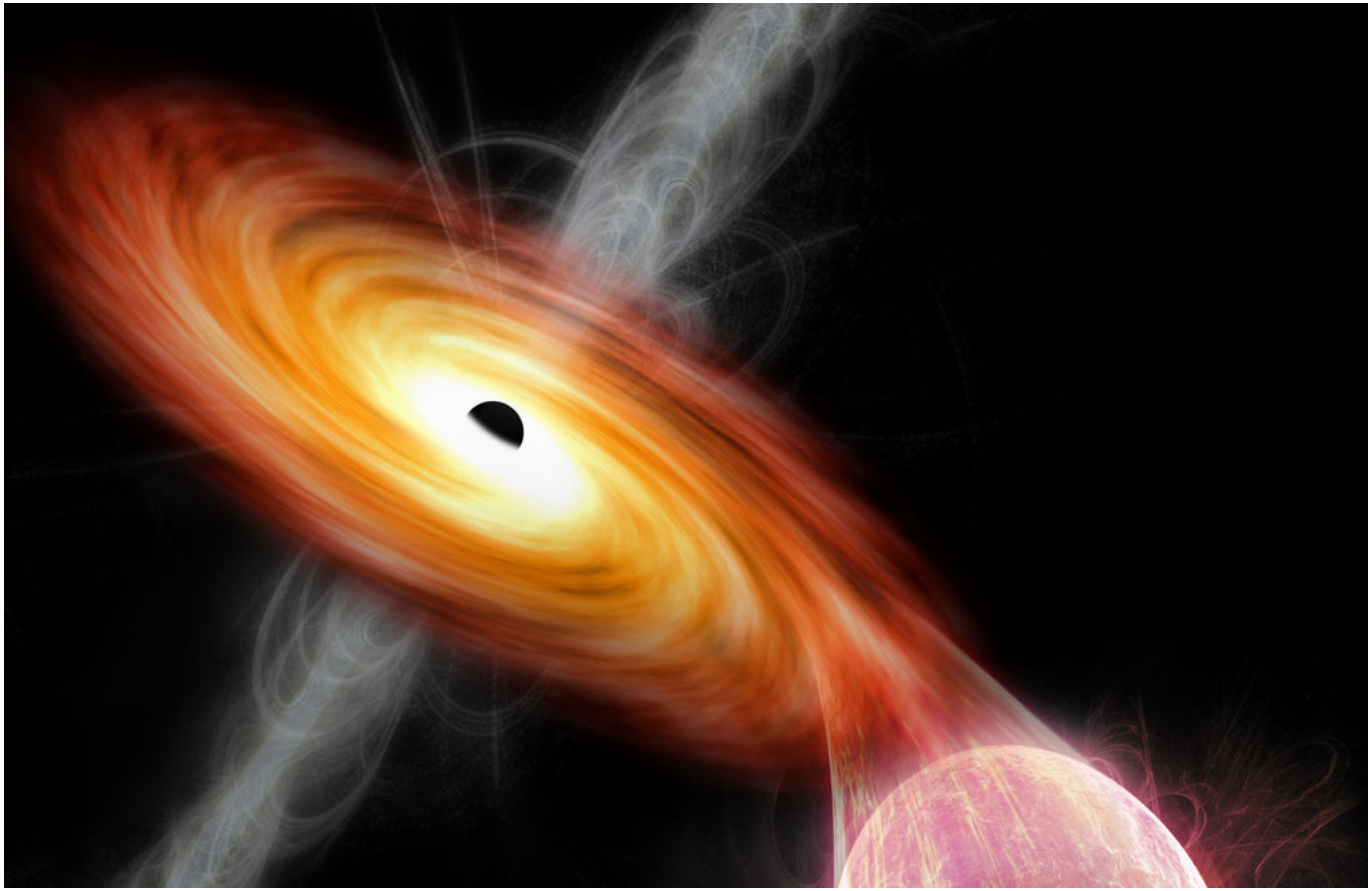
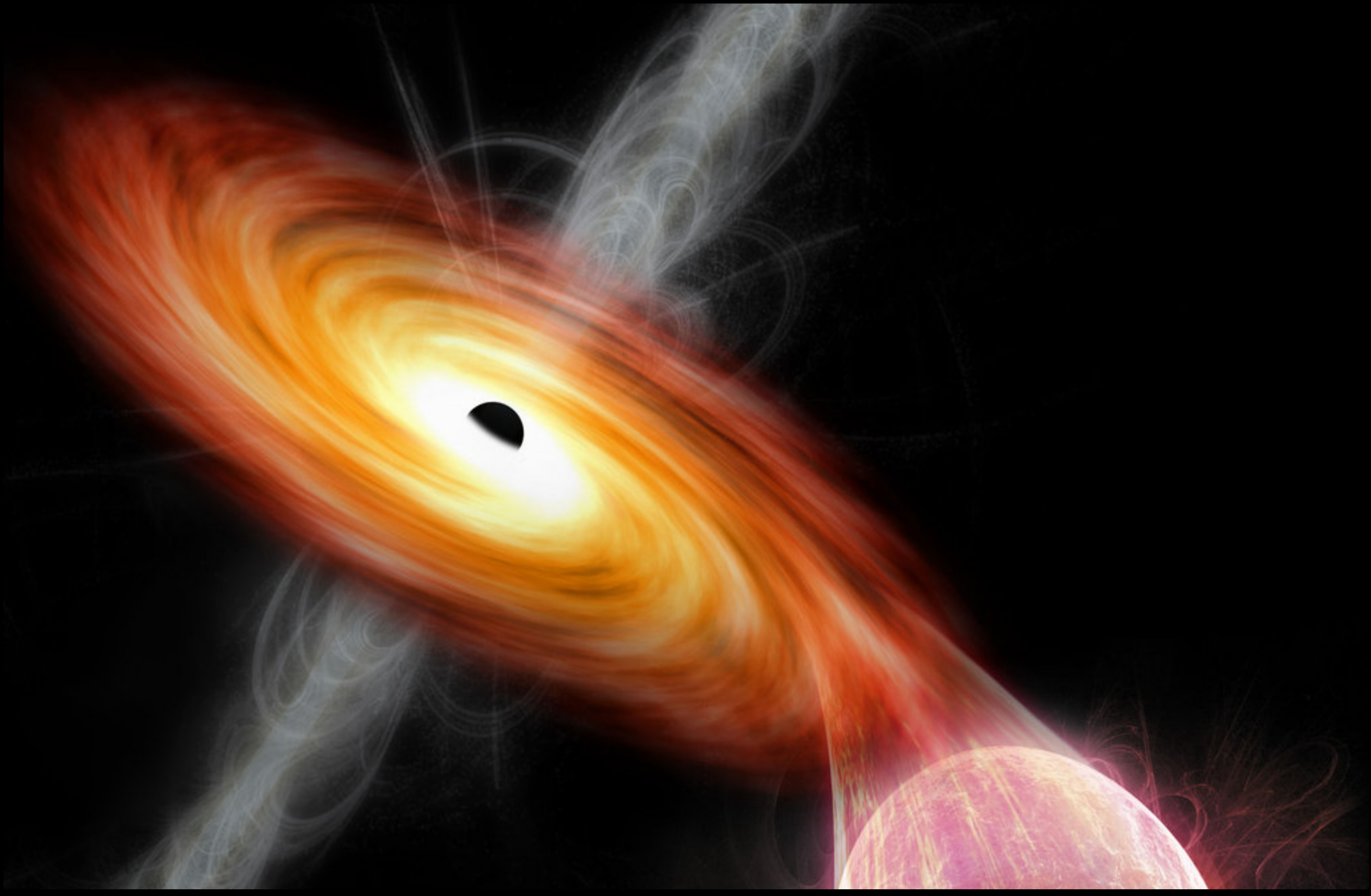
# High Energy Astrophysics

Dr. Adam Ingram

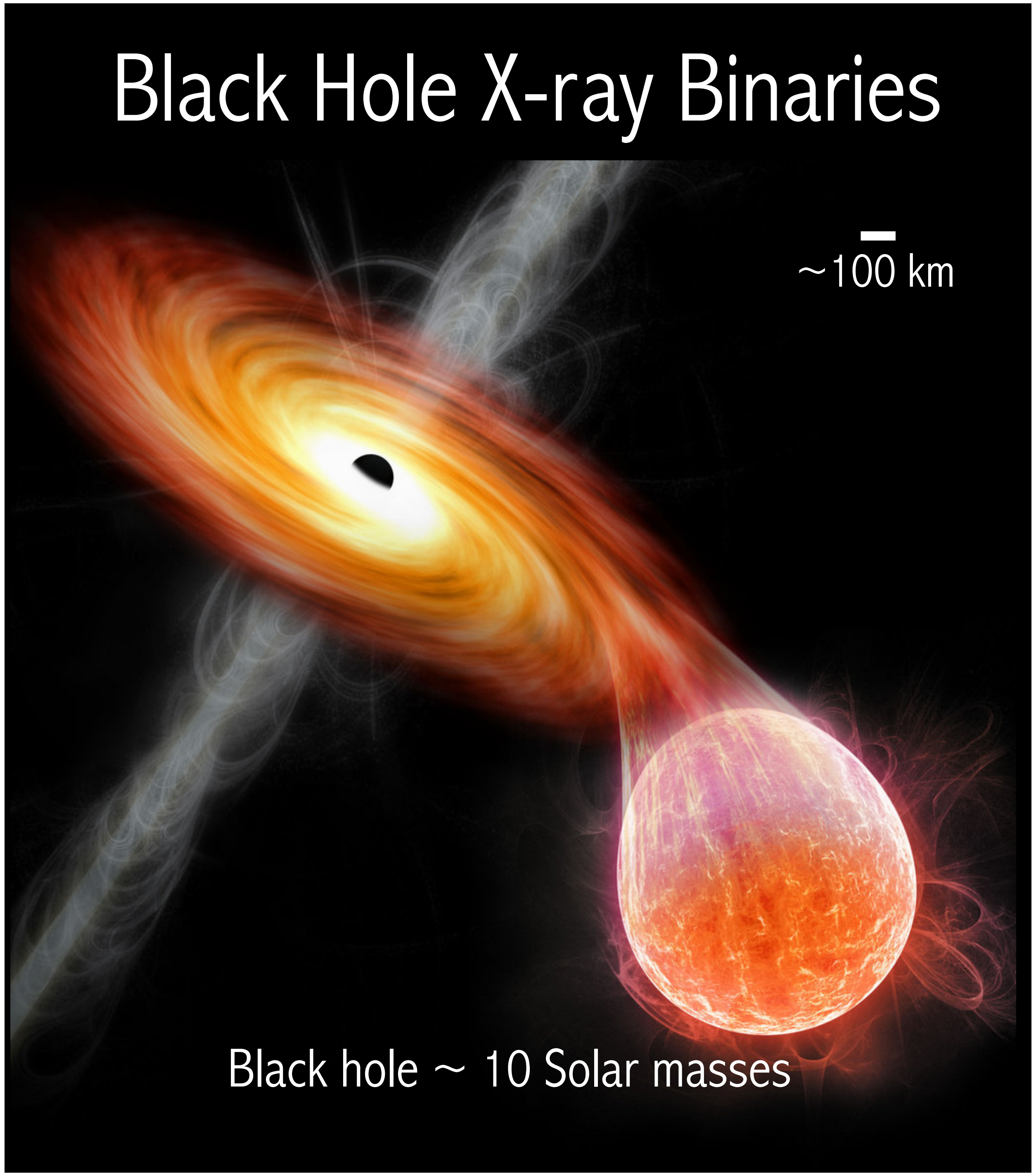


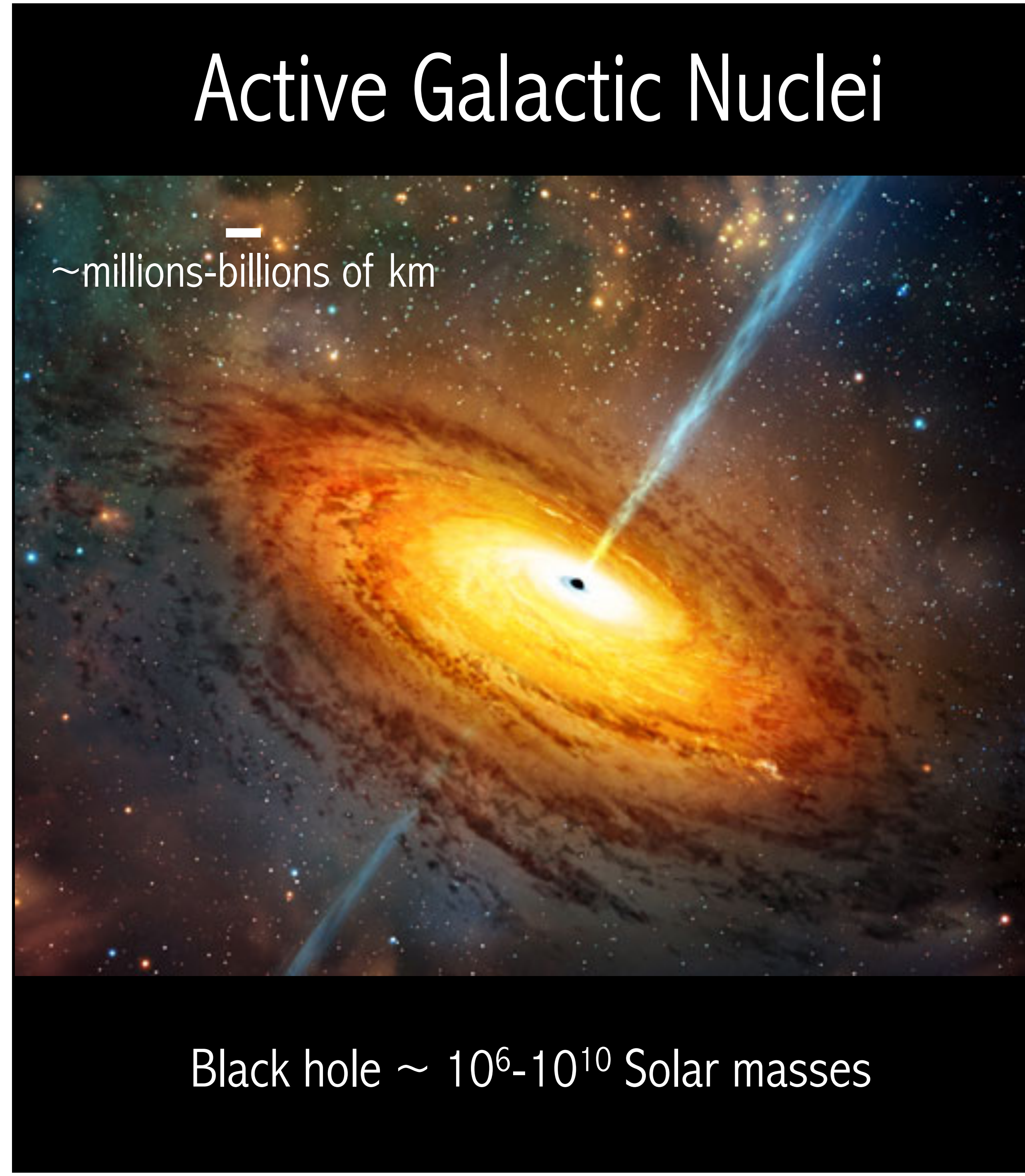
# Lecture 6

# Black Hole Accretion Discs



# Accreting Black Holes



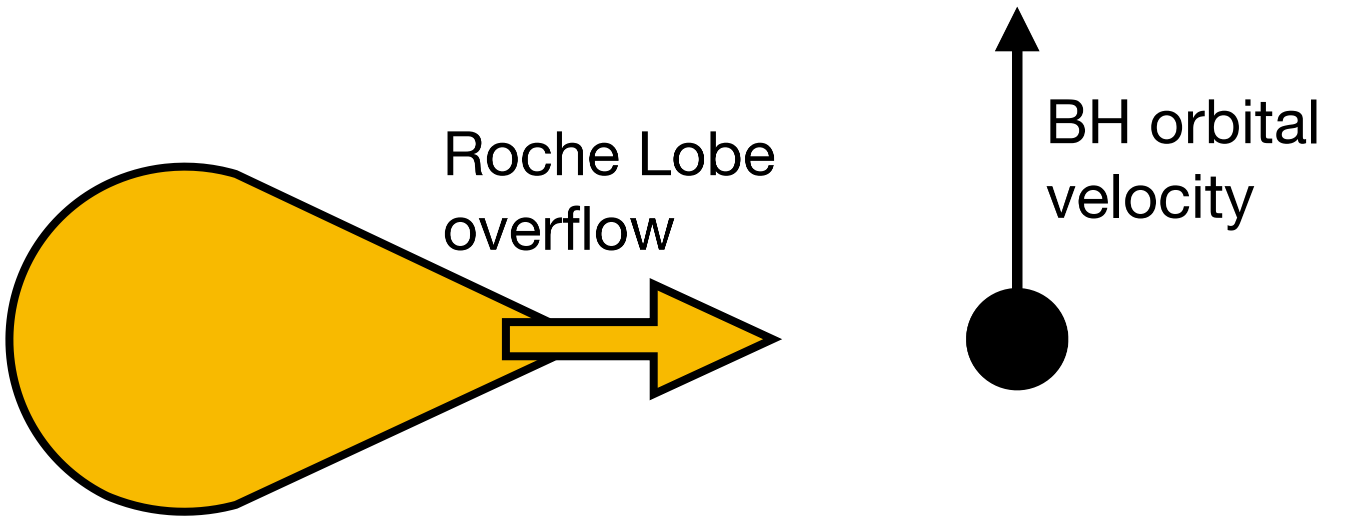




Power supply: gravitational potential energy of accreting material.

Therefore luminosity is the rest mass energy of accreted material multiplied by some efficiency factor: $L = \epsilon \dot{M} c^2$

# Disc formation

Companion star rest frame:



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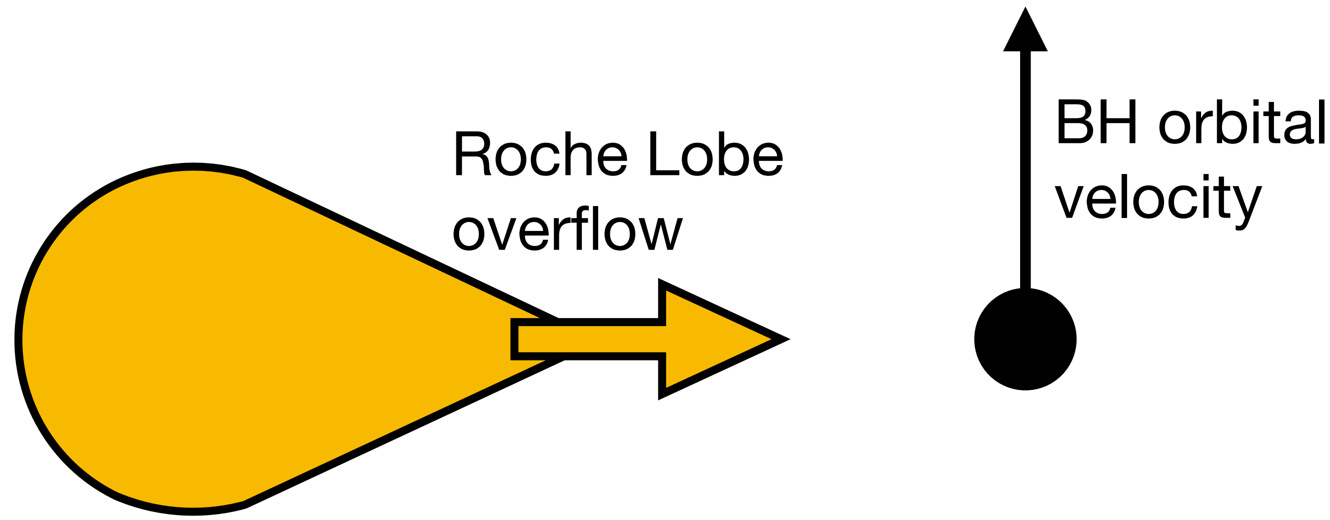


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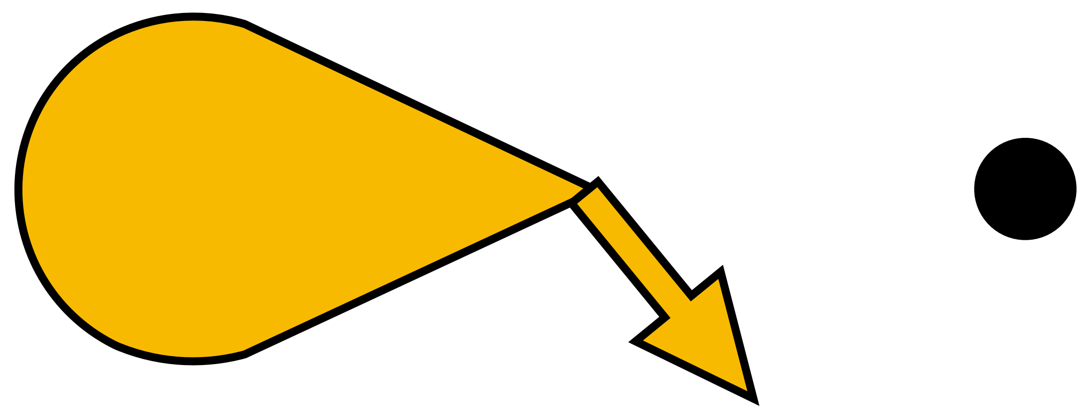

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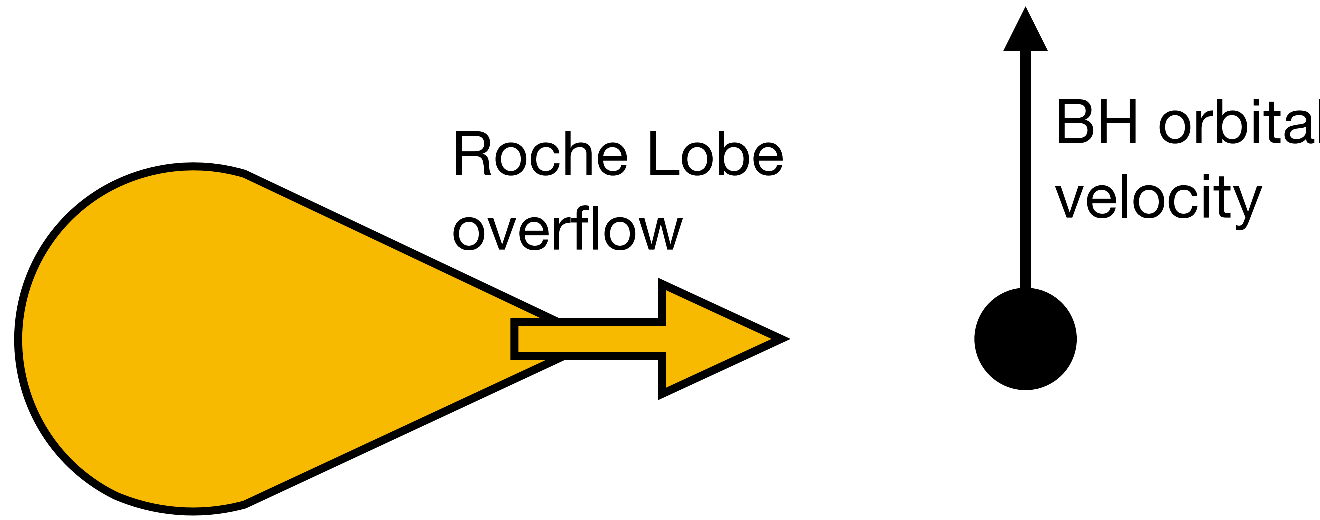


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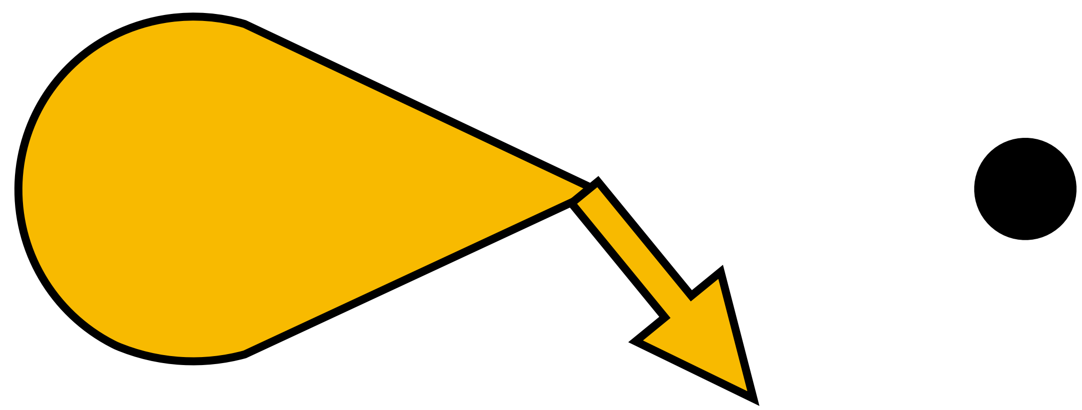
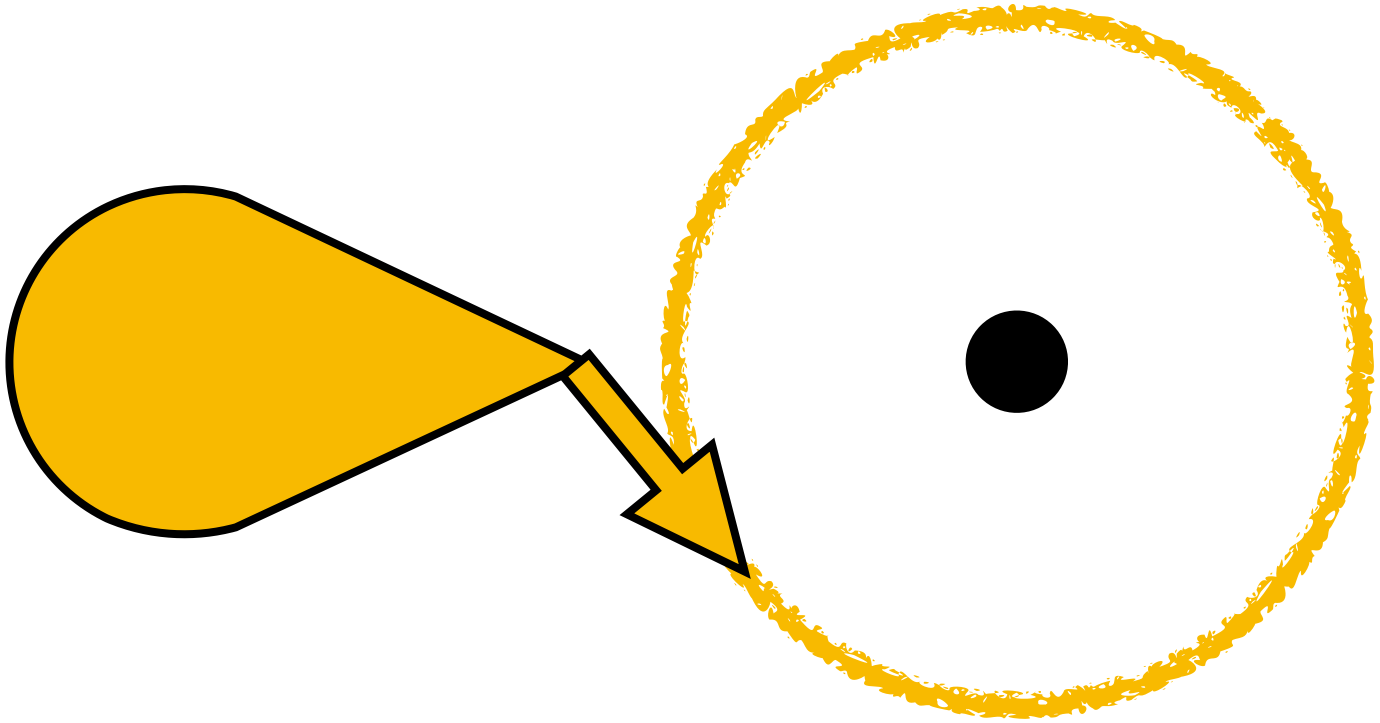


Therefore material has *angular momentum* about the black hole (similar arguments hold for AGN).

# Disc formation

- Orbit quickly circularises via collisions (circular orbit is minimum energy configuration).



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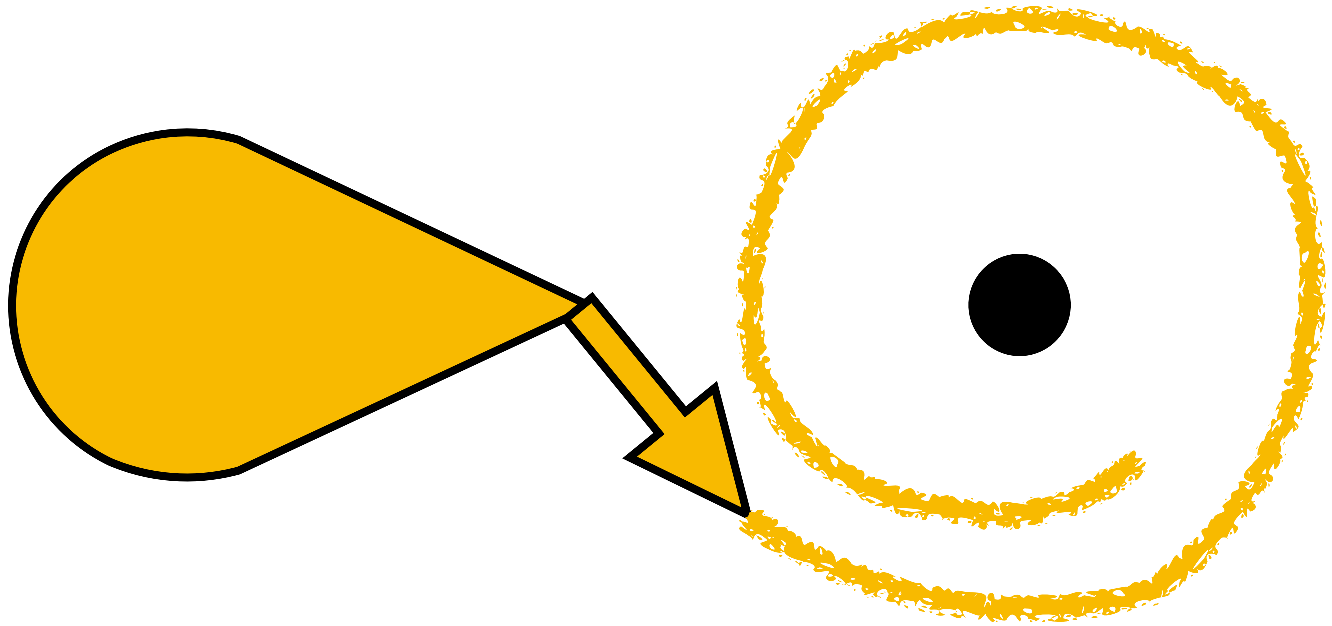

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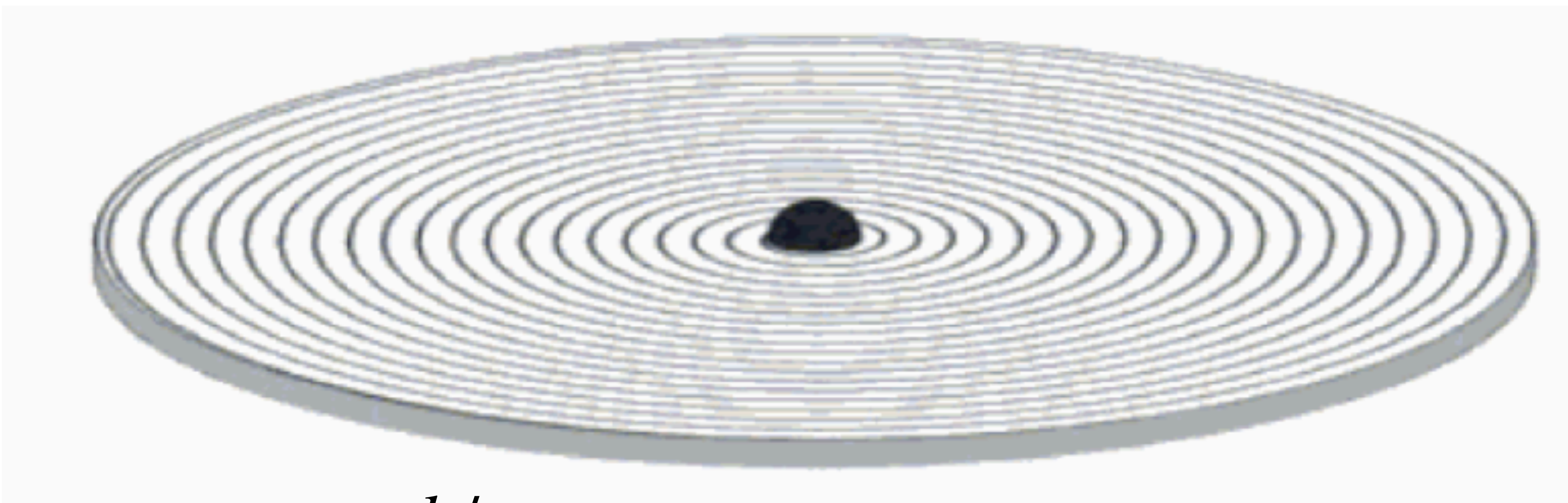
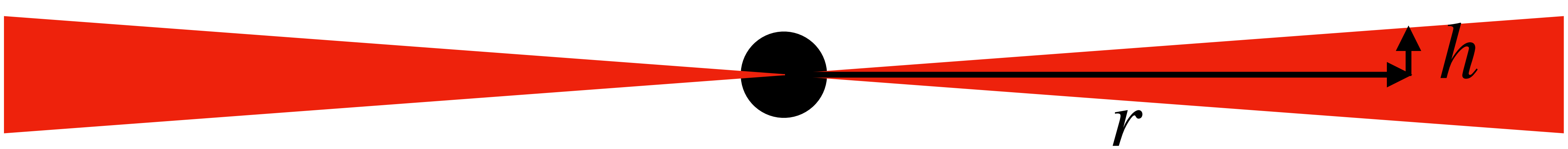


- Disc scale height: $h/r \sim \text{constant}$



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- Therefore *half* of the GPE liberated can be radiated away (virial theorem). Luminosity of annulus is:
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- This is *enormous*! Nuclear fusion has efficiency $\epsilon \approx 0.007$

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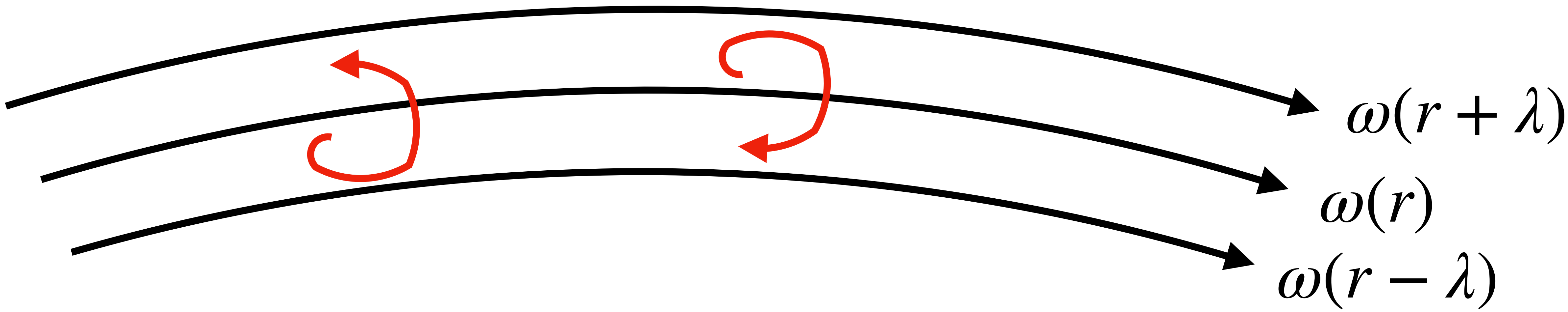


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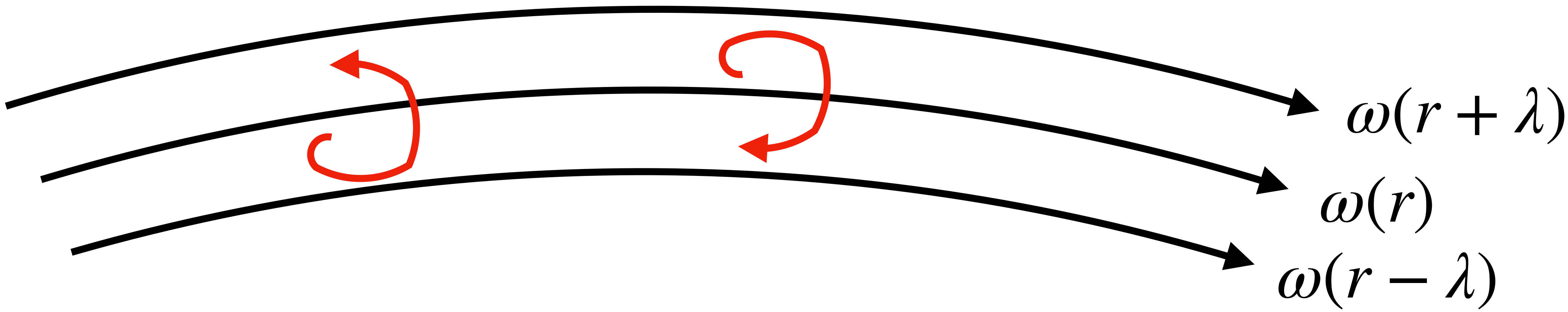


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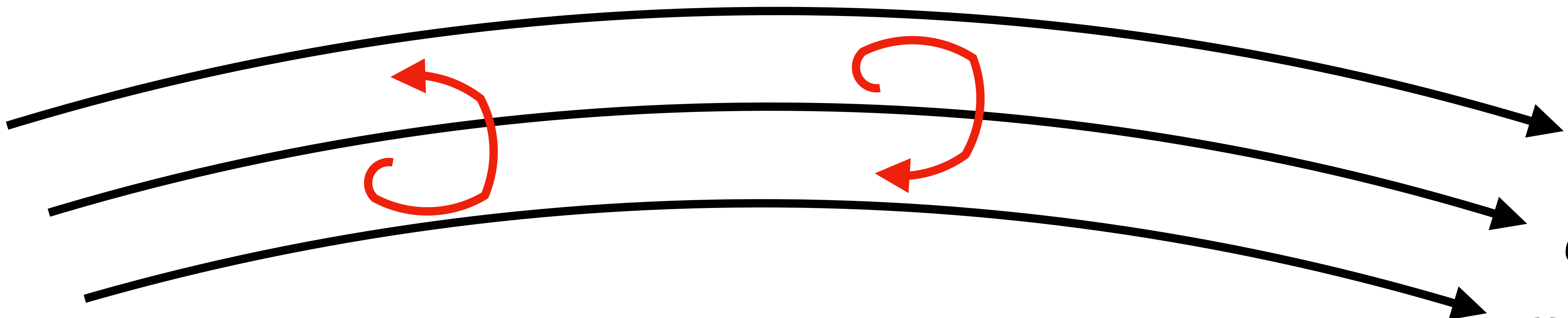


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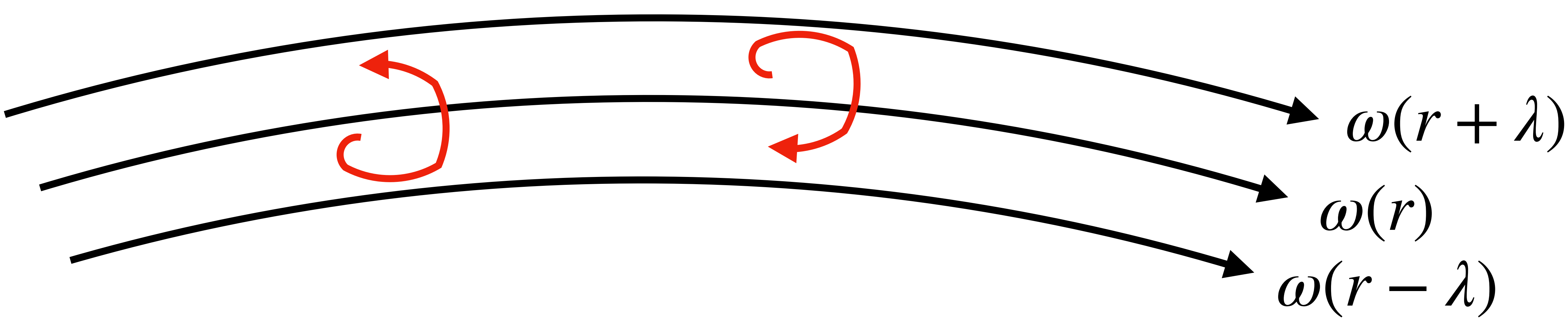
- In other words: viscosity (~friction) *slows down* the inner ring and *speeds up* the outer ring
- i.e. angular momentum transported outwards, material can spiral towards the BH.

# Angular Momentum Transport

So what is causing the turbulent motions (~viscosity)?



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- For viscosity due to random thermal motions of particles:

  $\bar{v} \sim c_s$ = sound speed

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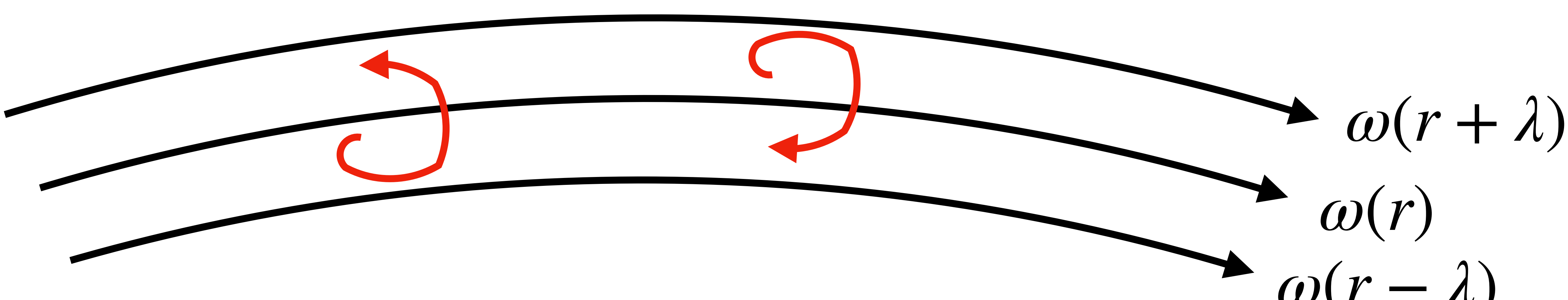
- The calculation is lengthy (see Longair or FKR) but it turns out that the mean free path is ***far*** too small (by a factor ~$10^{12}$) to provide the required viscosity.



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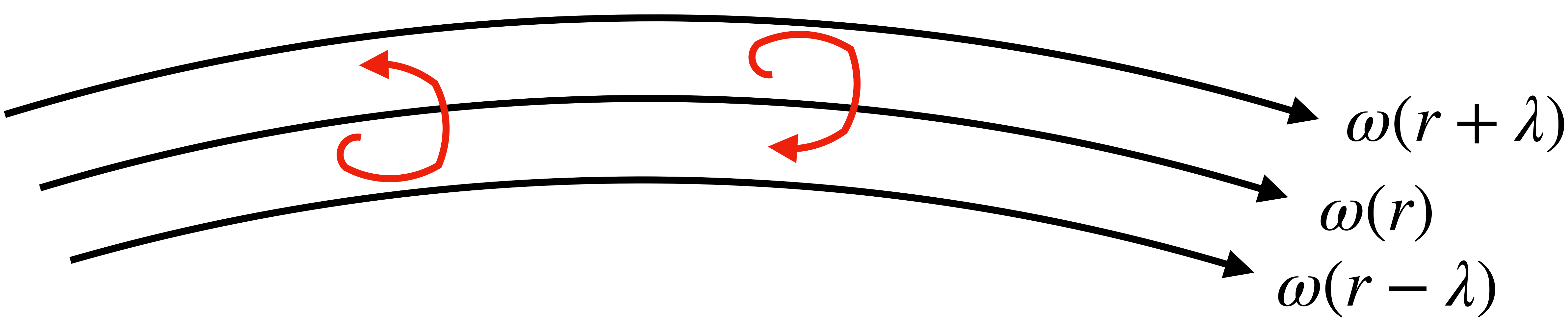

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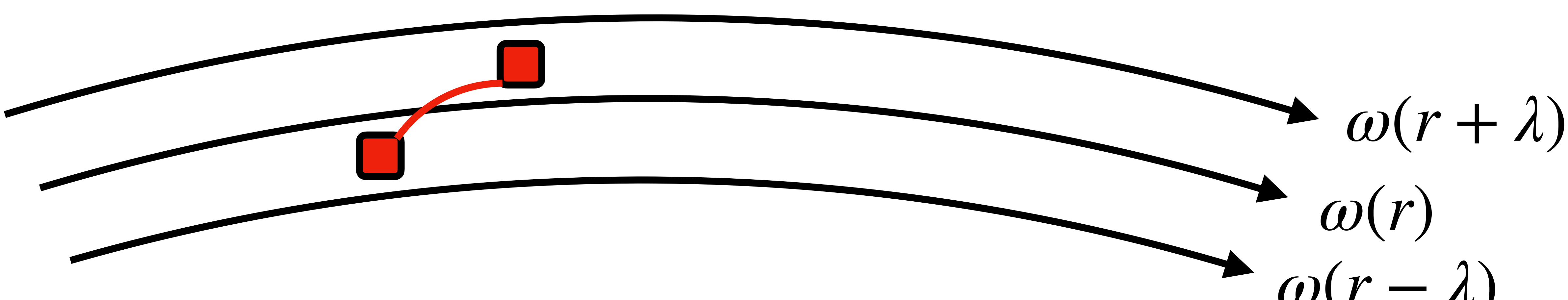

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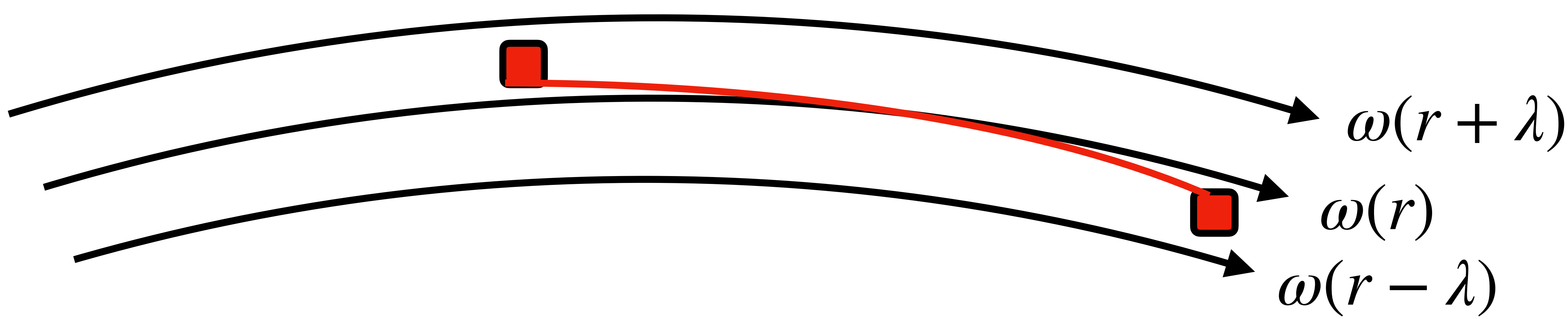

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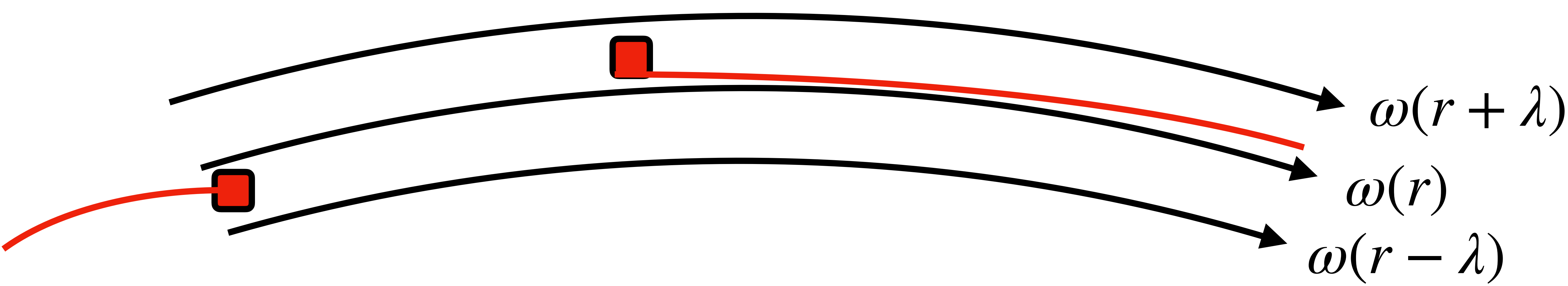
- Effective viscosity must be provided by *hydrodynamic turbulence*.
- The magneto-rotational instability provides the required viscosity: B-field lines connect parcels of gas, differential rotation stretches distance between these parcels, causes field lines to become tangled and generate turbulence. Like winding an elastic band round and round the disc.



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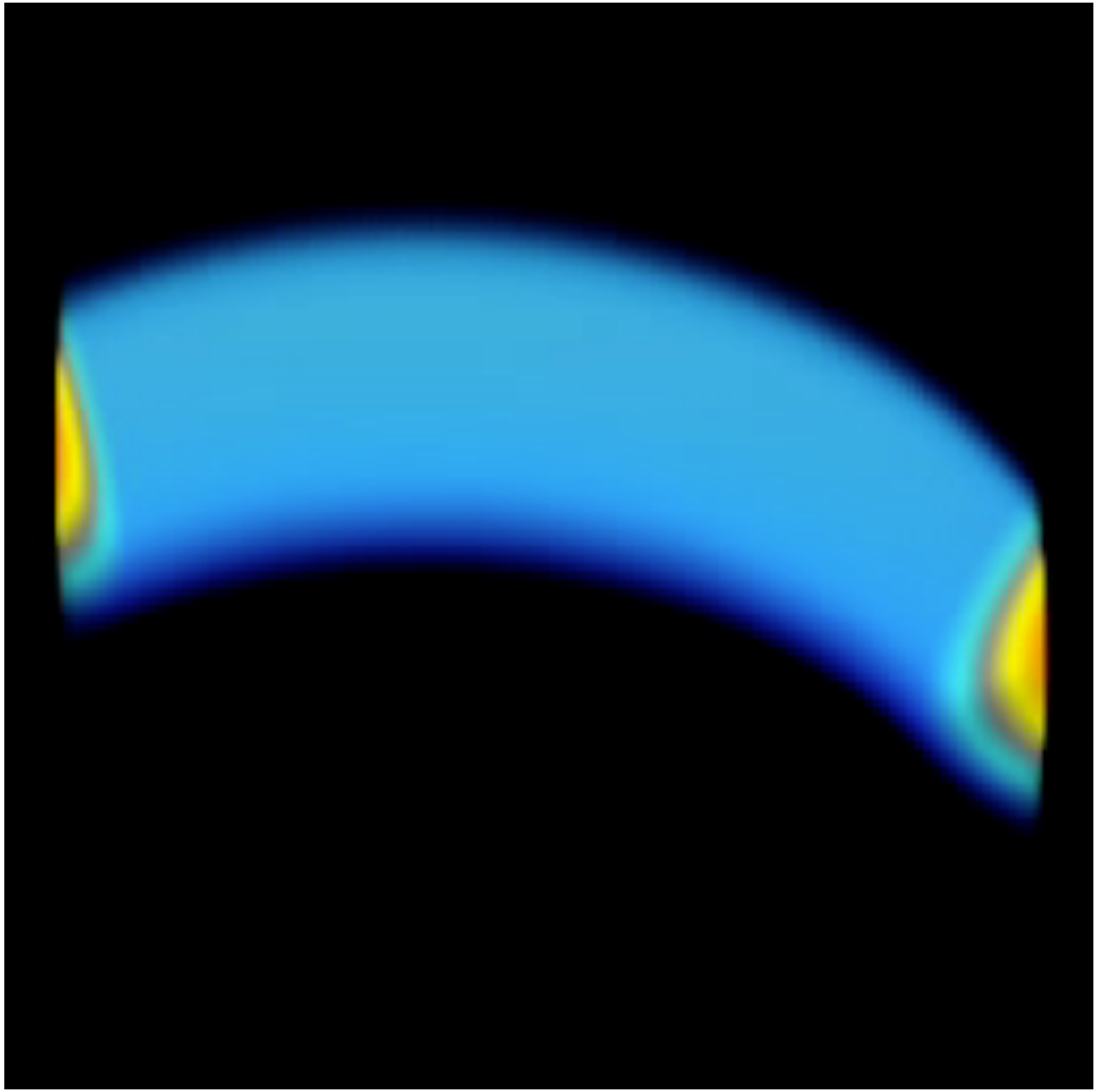

Krolik, de Villiers & Hawley

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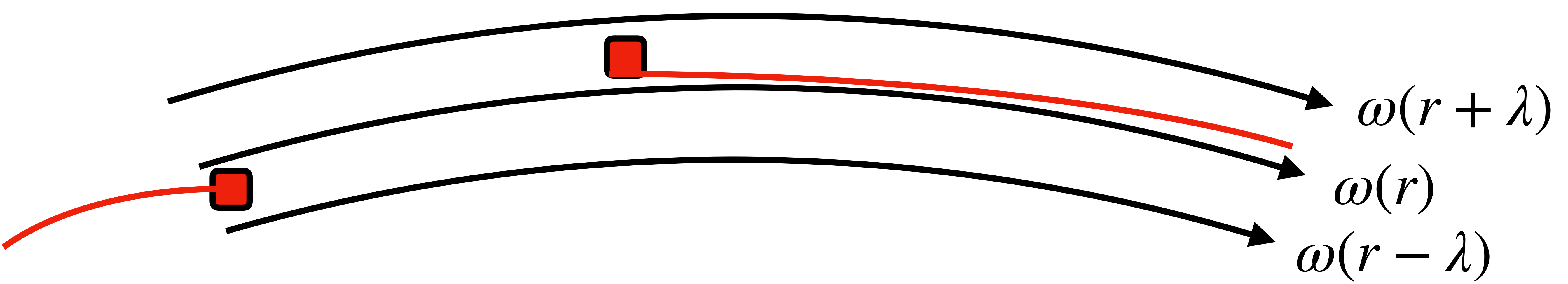
- Shakura & Sunyaev (1973) famously assumed for the *kinematic viscosity*:

$$\nu = \alpha c_s h$$

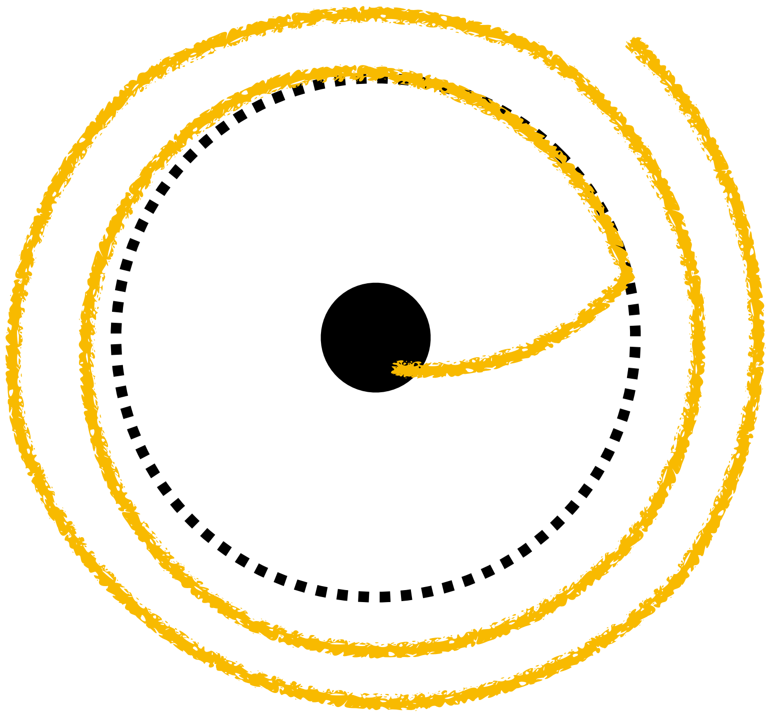
- $\alpha$ is a dimensionless constant: speed of eddies $\lesssim c_s$ ; length scale of eddies $\lesssim h$ , therefore $\alpha \lesssim 1$ .

- The "alpha-disc" model has been remarkably successful.



# What happens at $r_{isco}$?

- Inside of ISCO, no stable orbits so material is in free fall.



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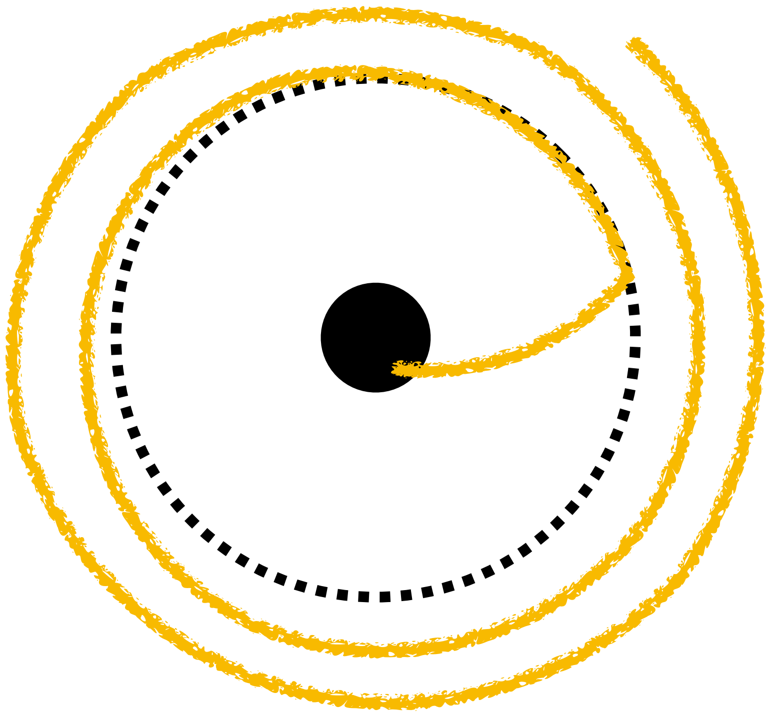

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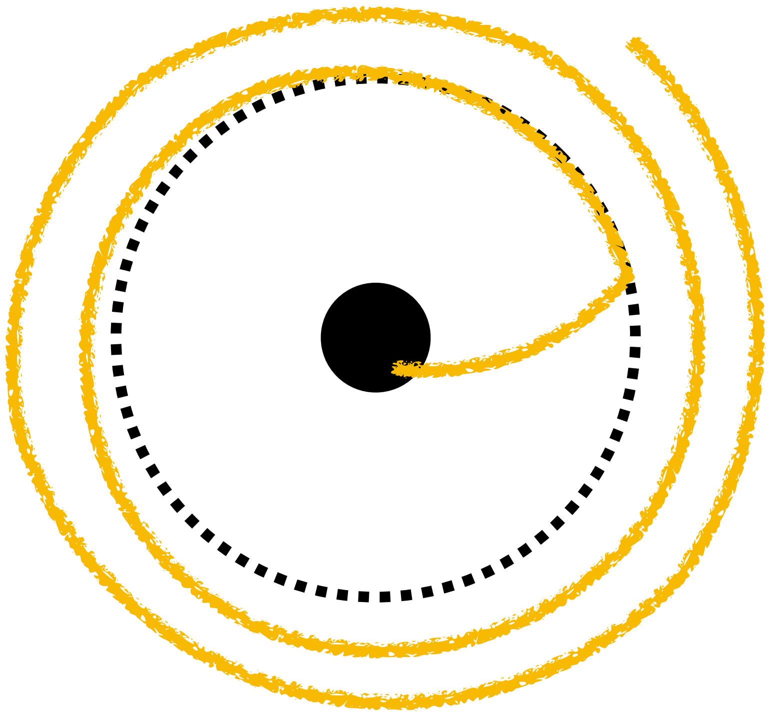

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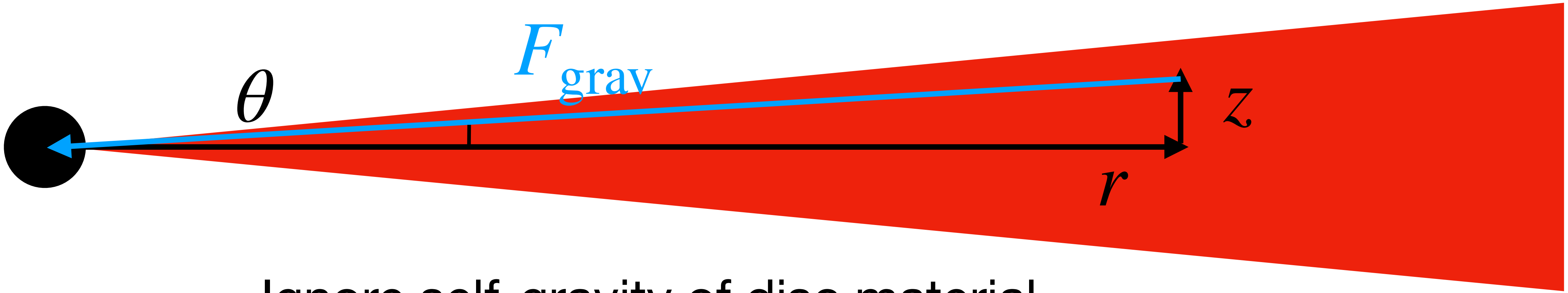
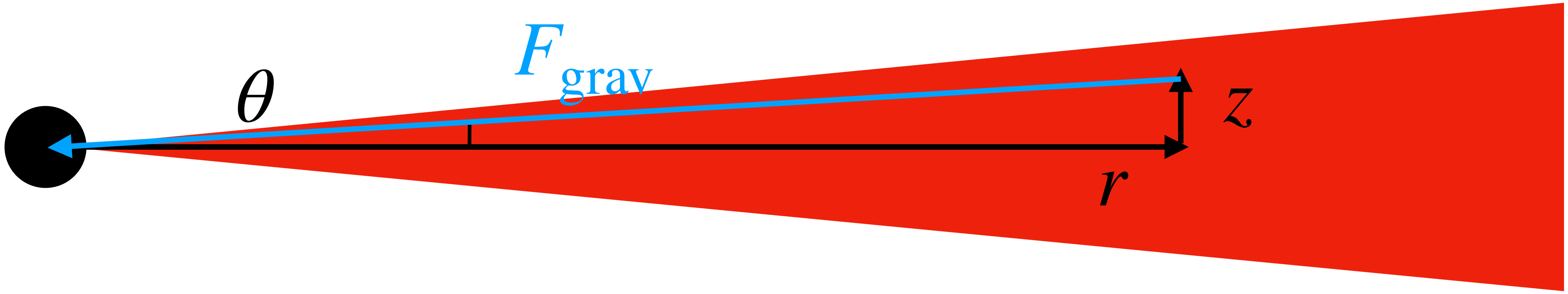
- Therefore lower spin supermassive black holes can presumably grow faster than higher spin ones.

# Disc vertical structure




Ignore self-gravity of disc material

# Disc vertical structure




- Disc in hydrostatic equilibrium, therefore vertical pressure gradient:

$$\frac{dP}{dz} = -\rho g_z$$

$g_z$ is vertical component of gravitational acceleration, ρ is mass density

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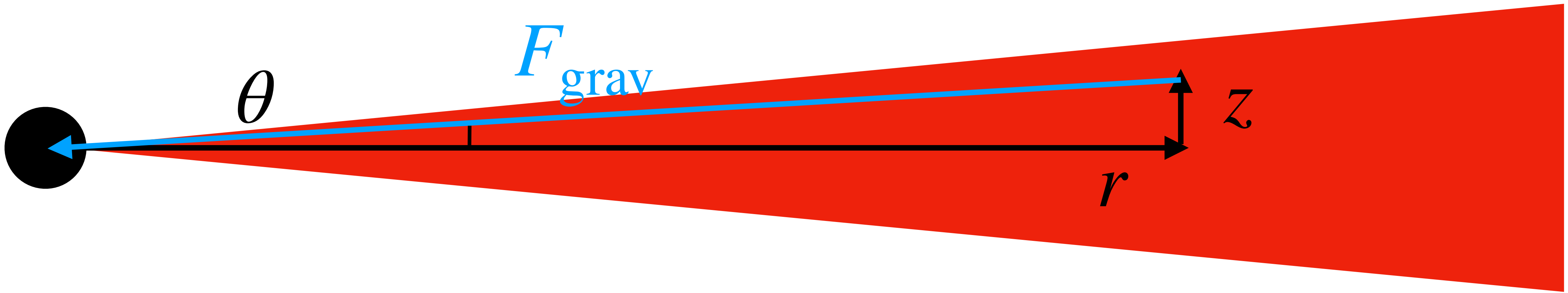


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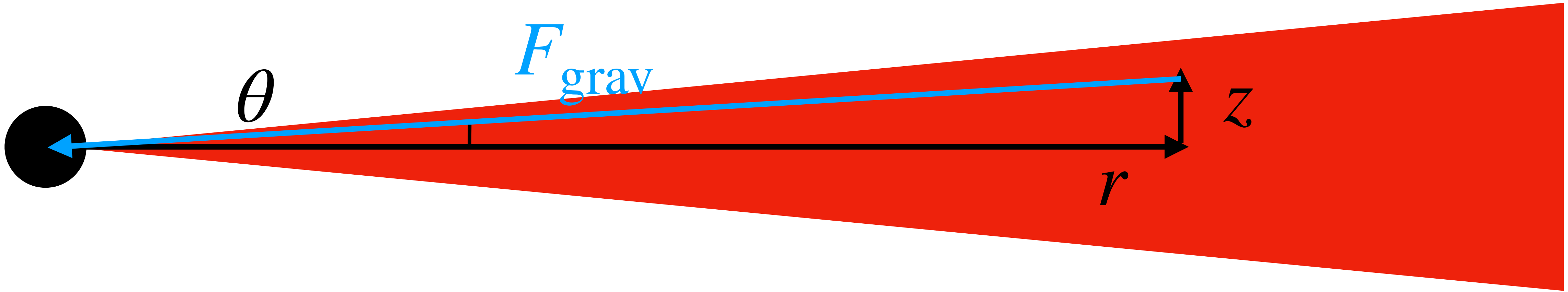


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(small angle approximation)

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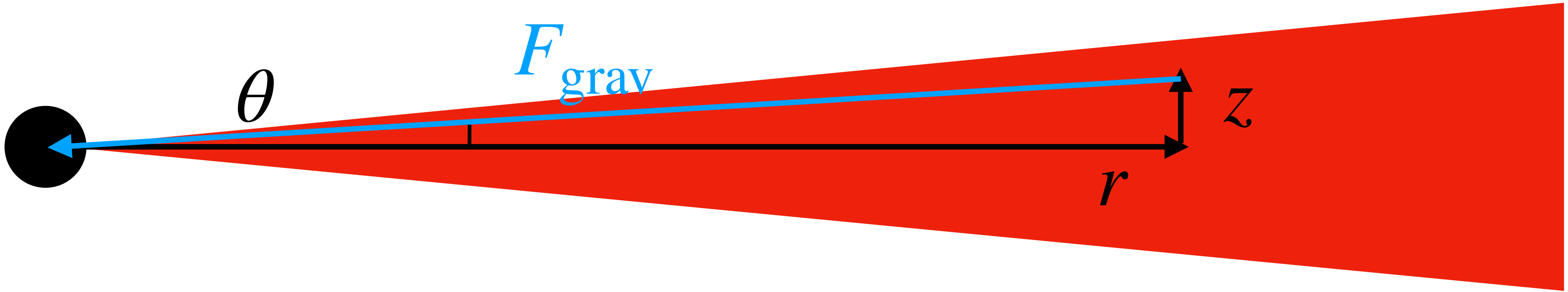


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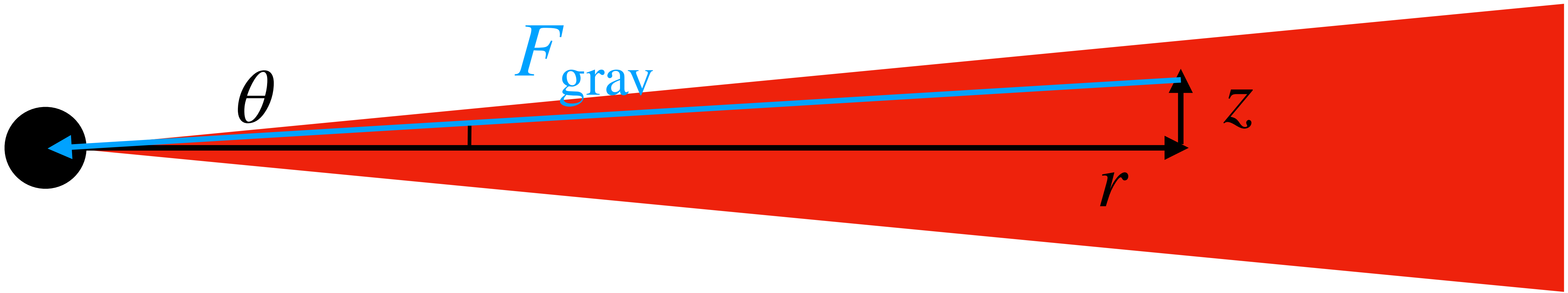


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- Therefore density is Gaussian with peak at z=0 and width h:

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$$\frac{h}{r} \ll 1 \implies$$ <u>*disc rotation is highly supersonic.*</u>

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- From Stefan-Boltzmann law, the temperature is:

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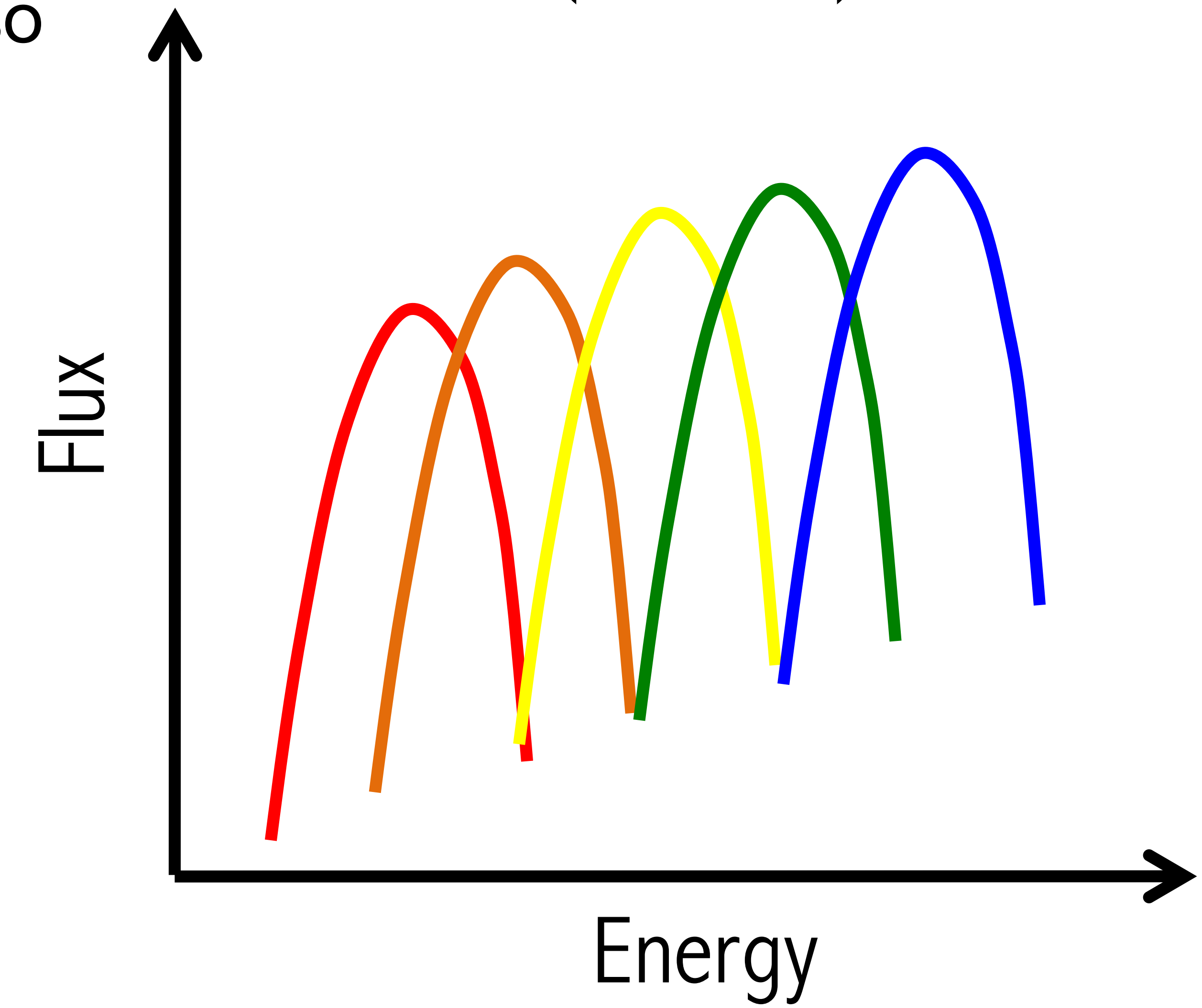


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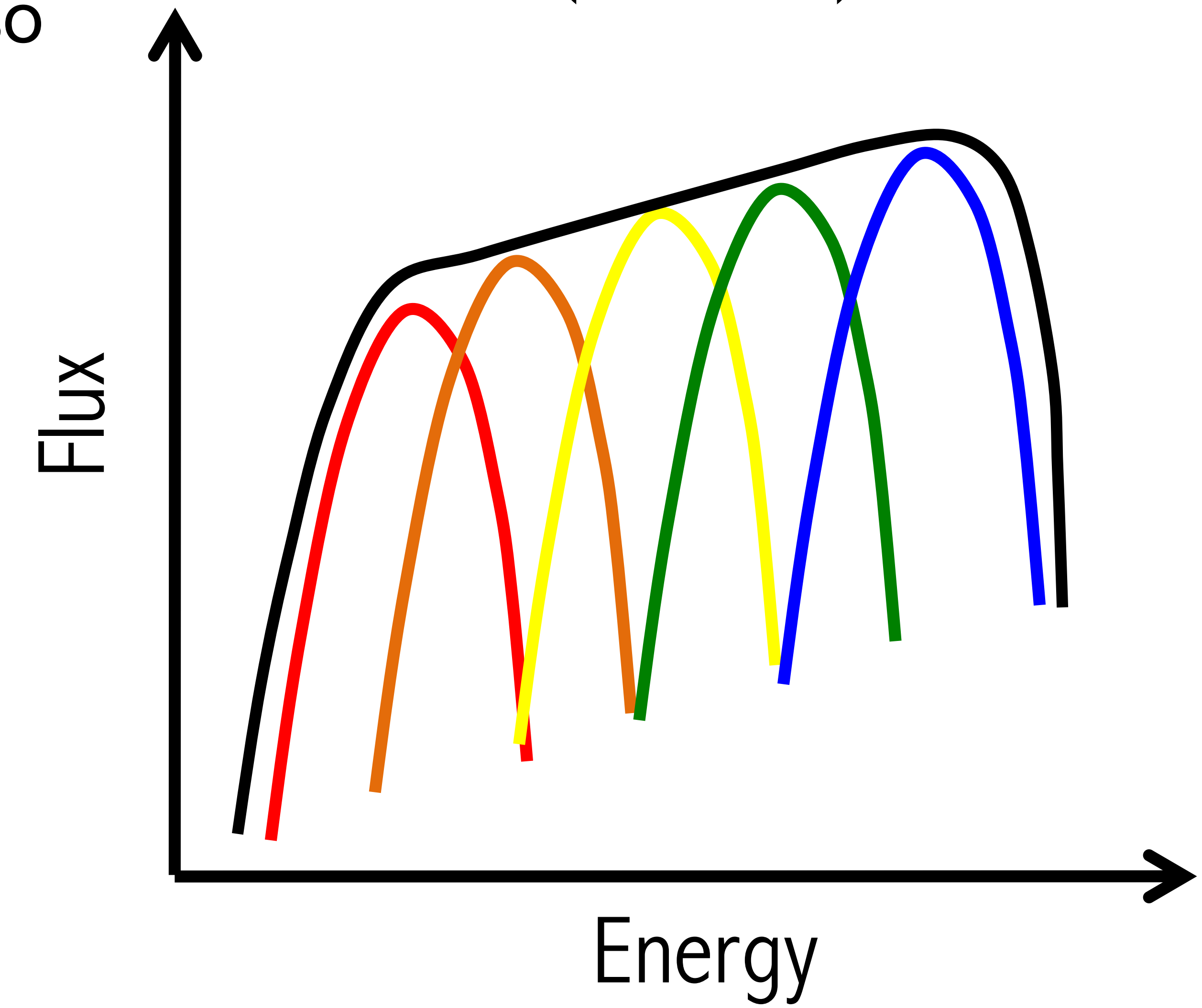


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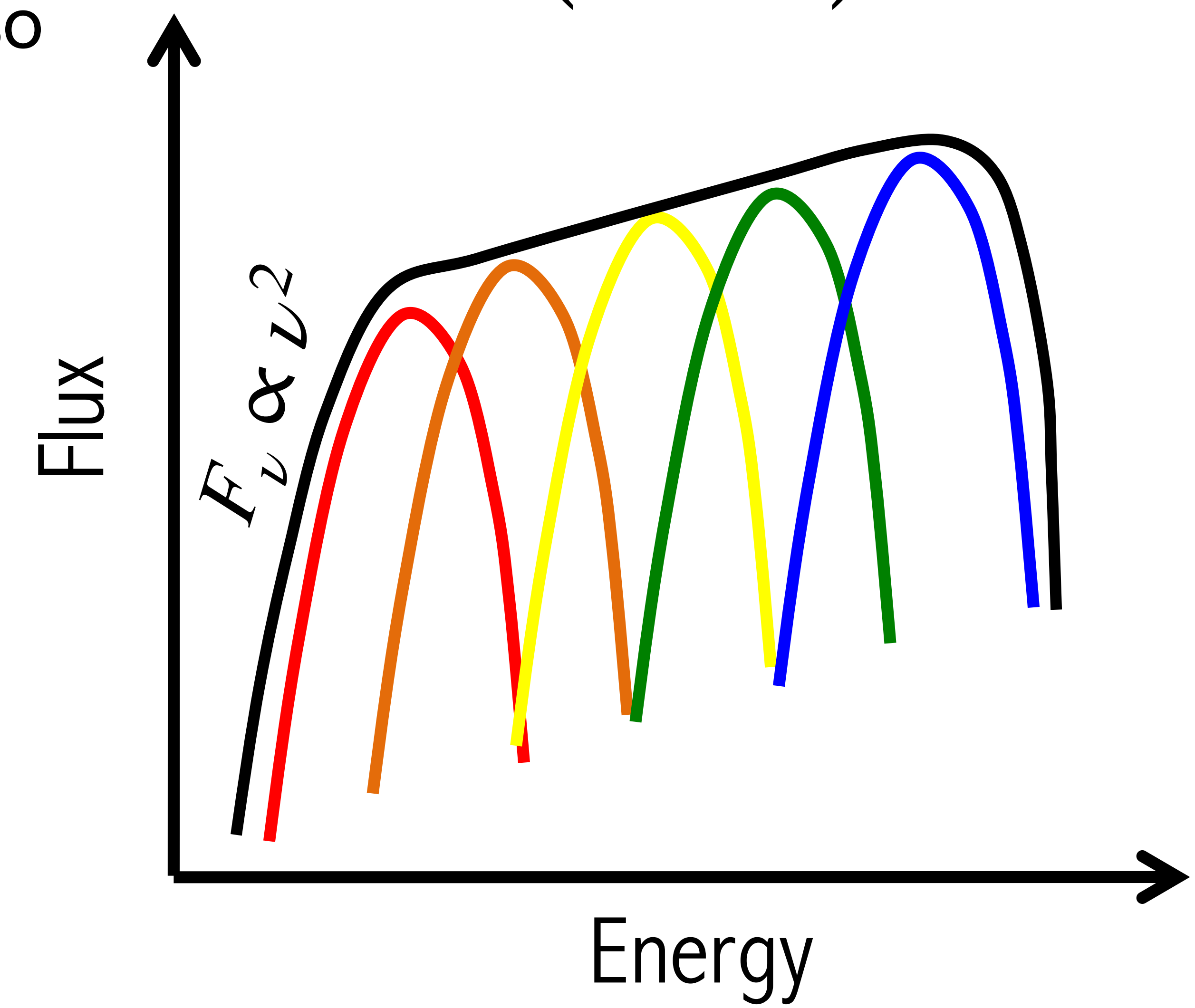


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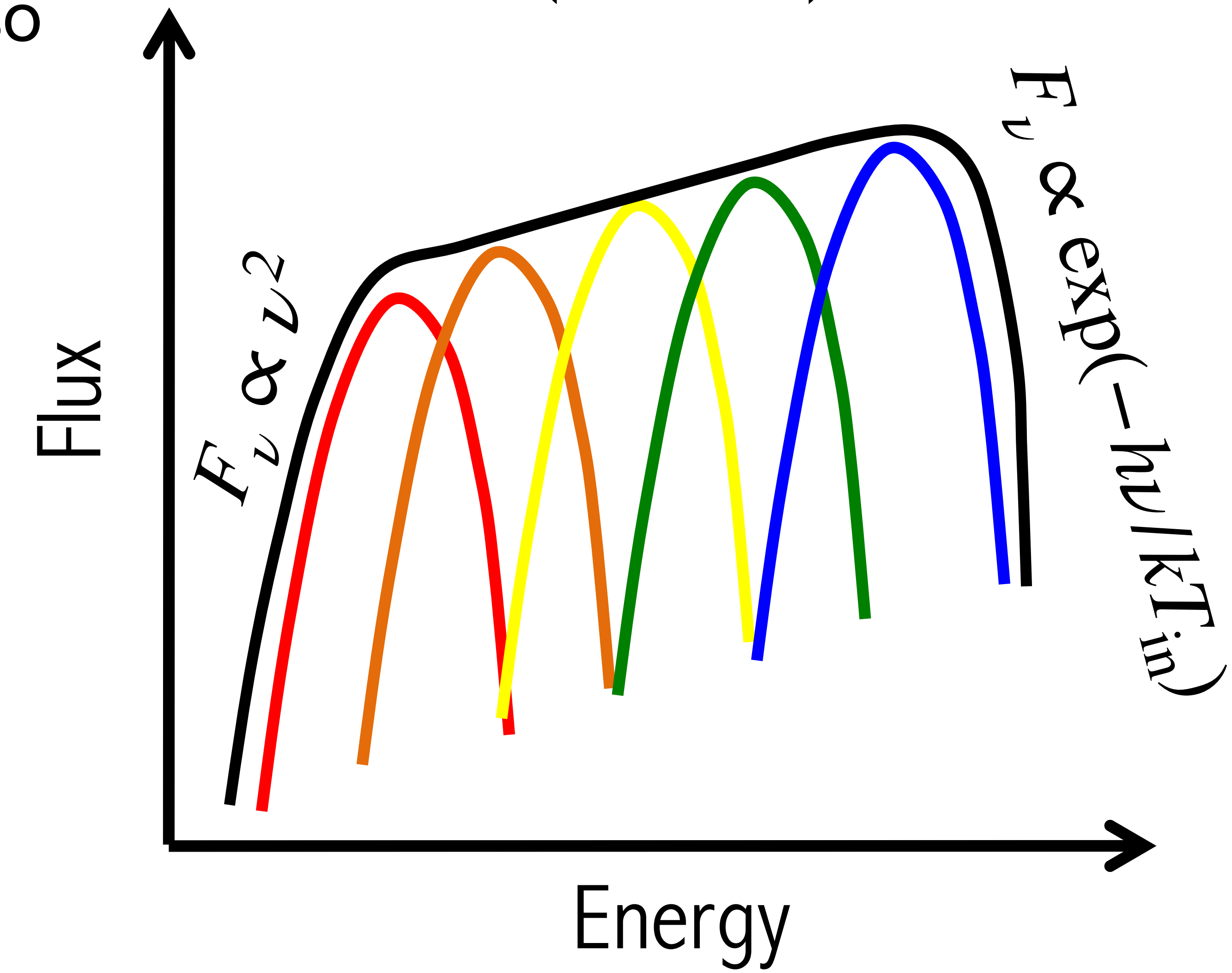
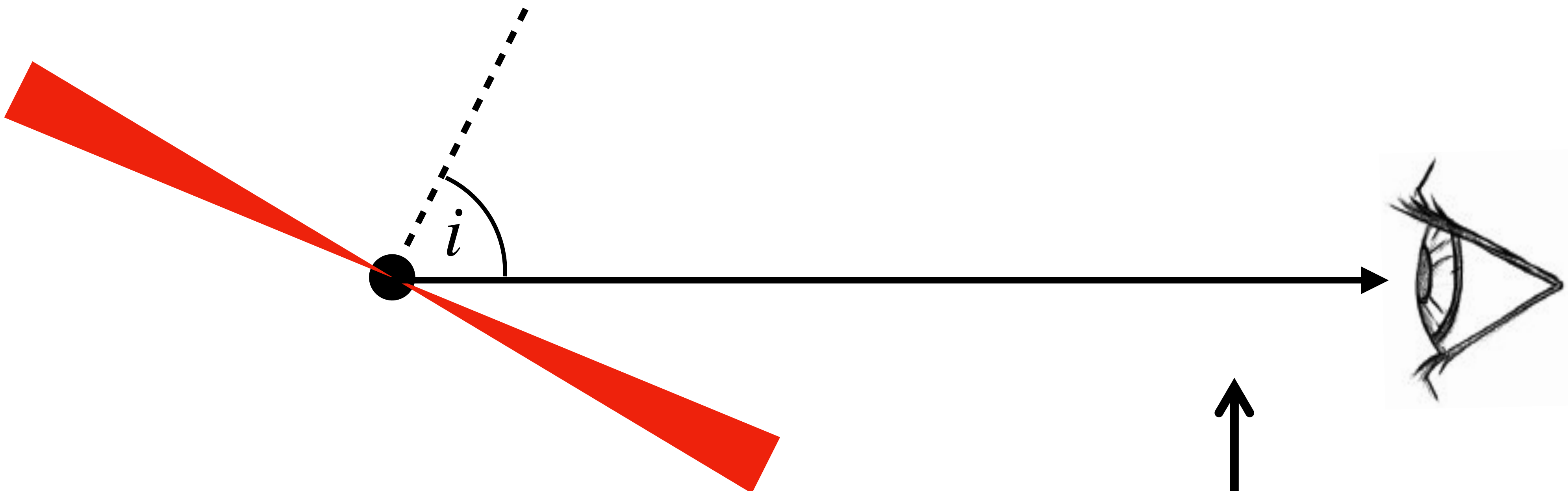
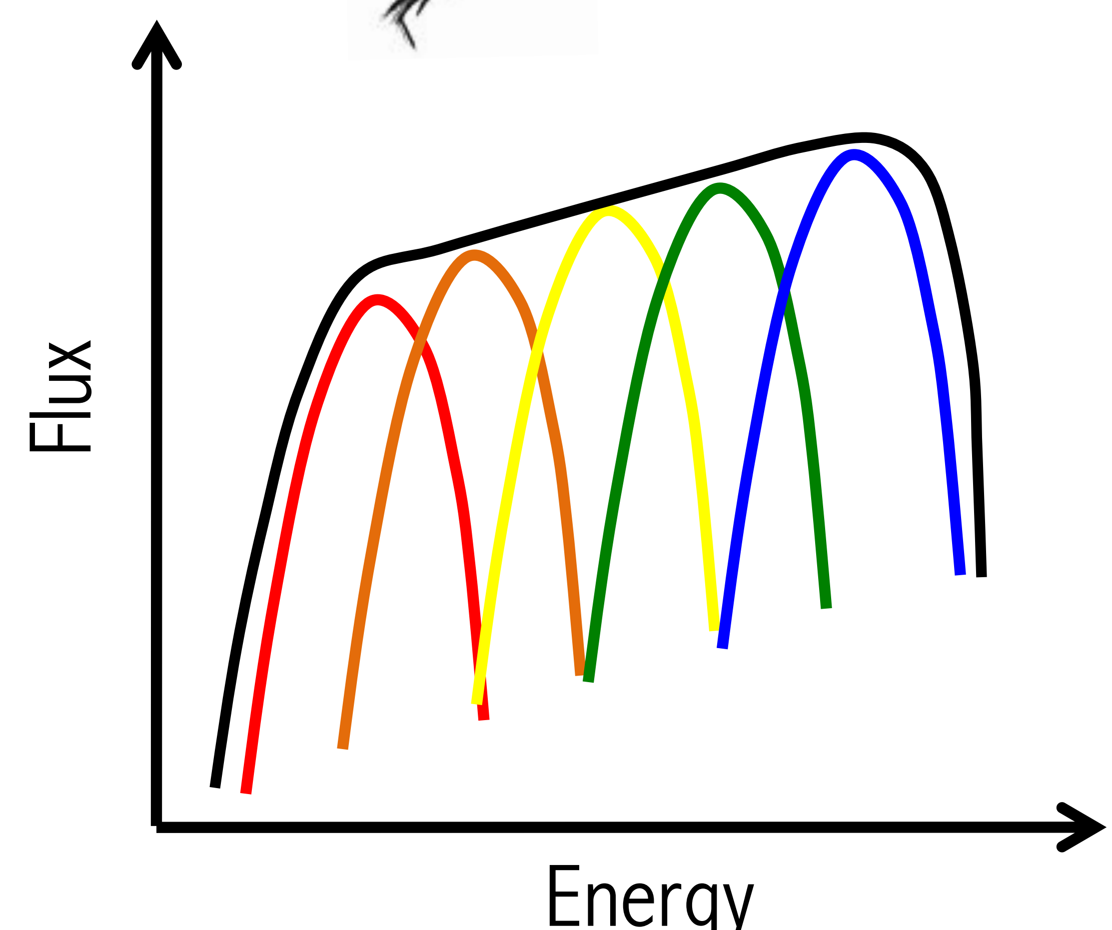



# Disc Spectrum

Intermediate frequencies:

$$F_\nu = \int_{r_{\rm in}}^{r_{\rm out}} B_\nu[T(r)]d\Omega(r) \approx \int_{r_{\rm in}}^{r_{\rm out}} B_\nu[T(r)]\frac{2\pi r\cos i\ dr}{D^2}$$



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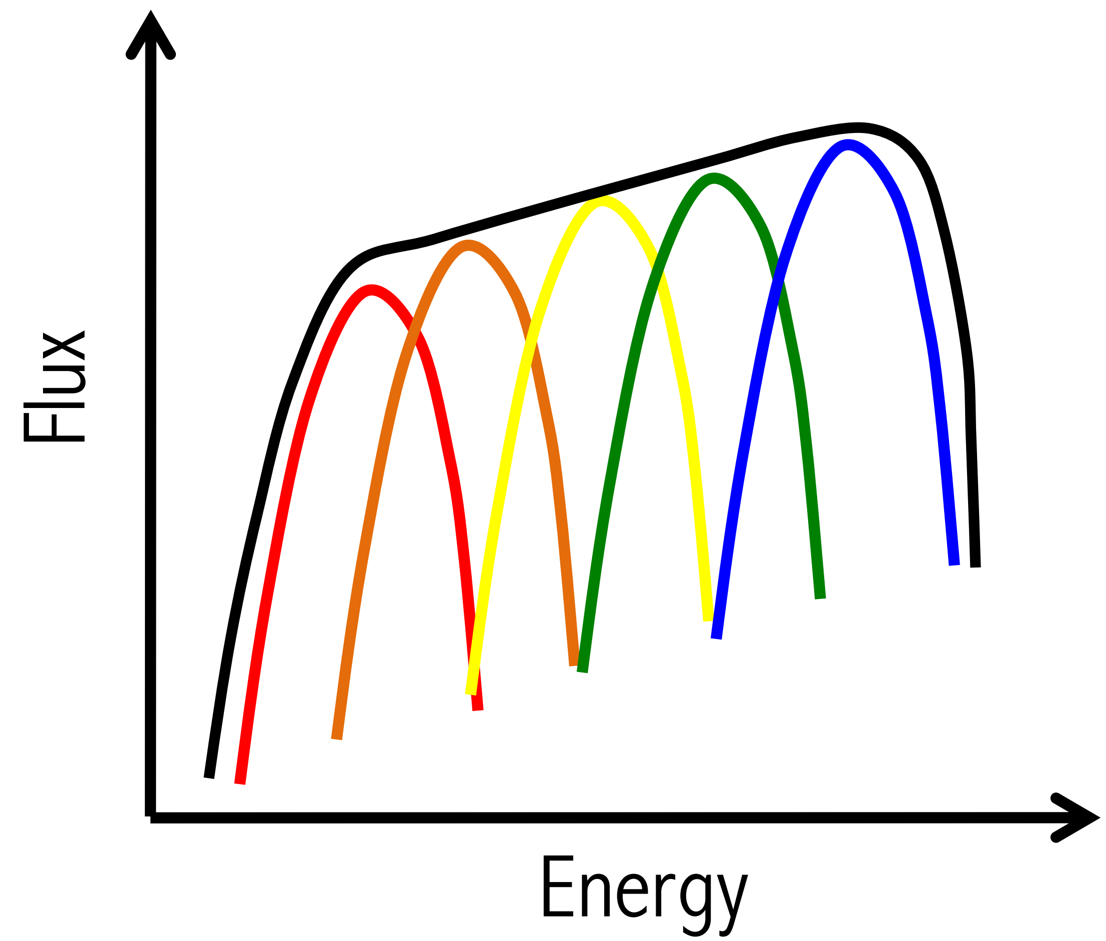

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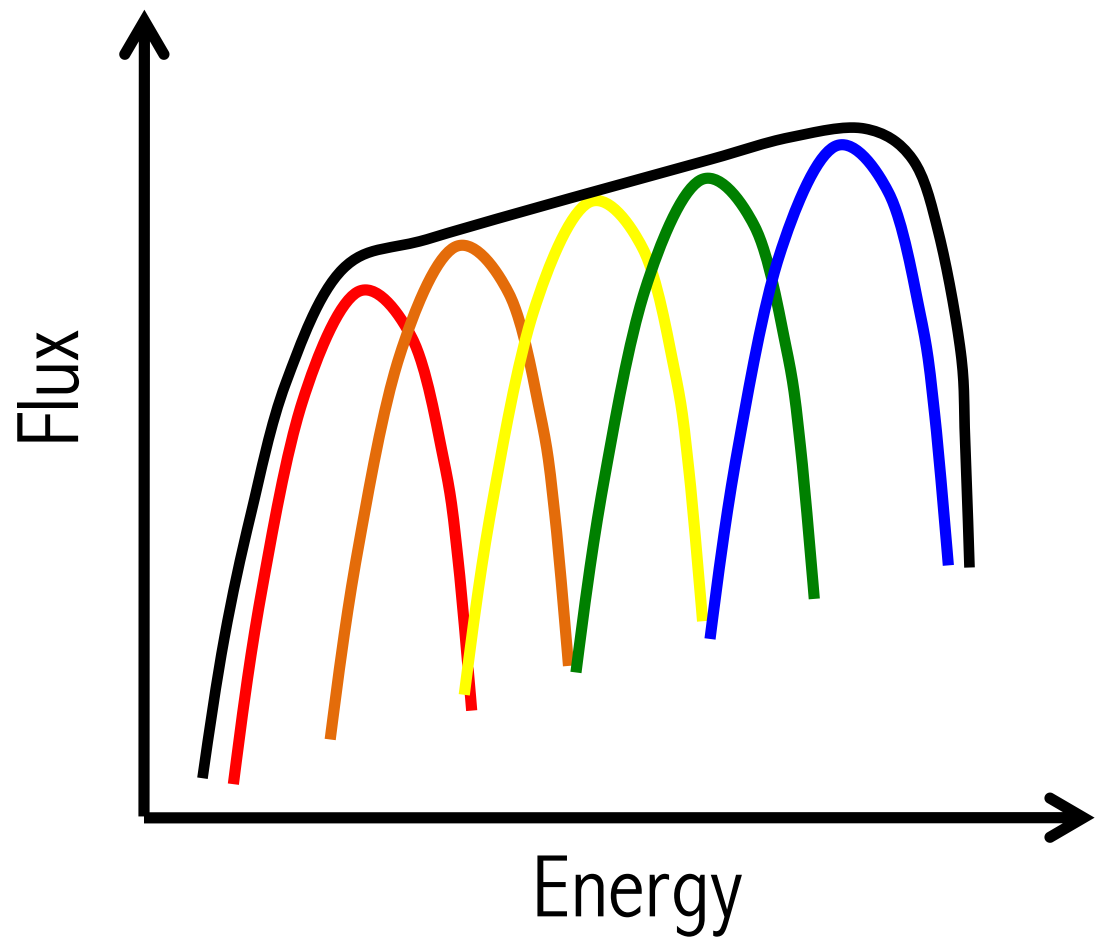

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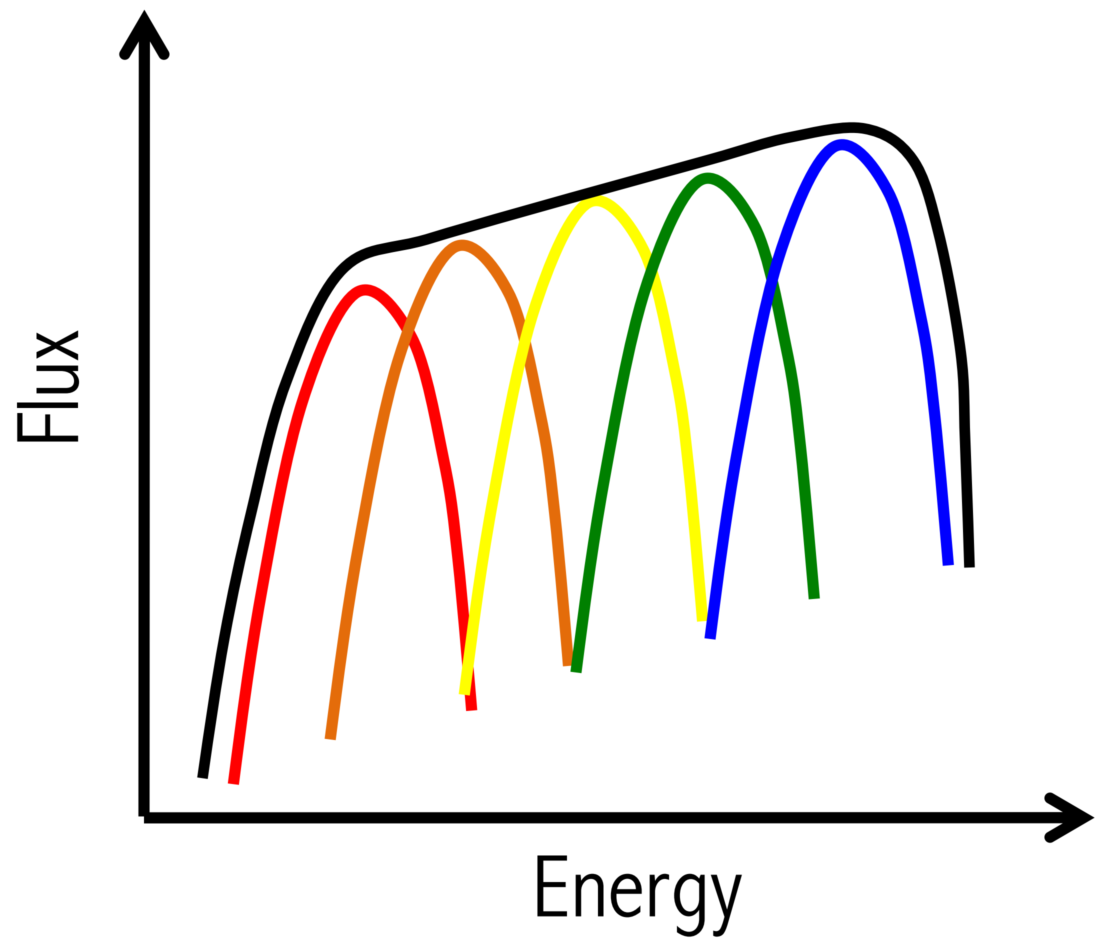

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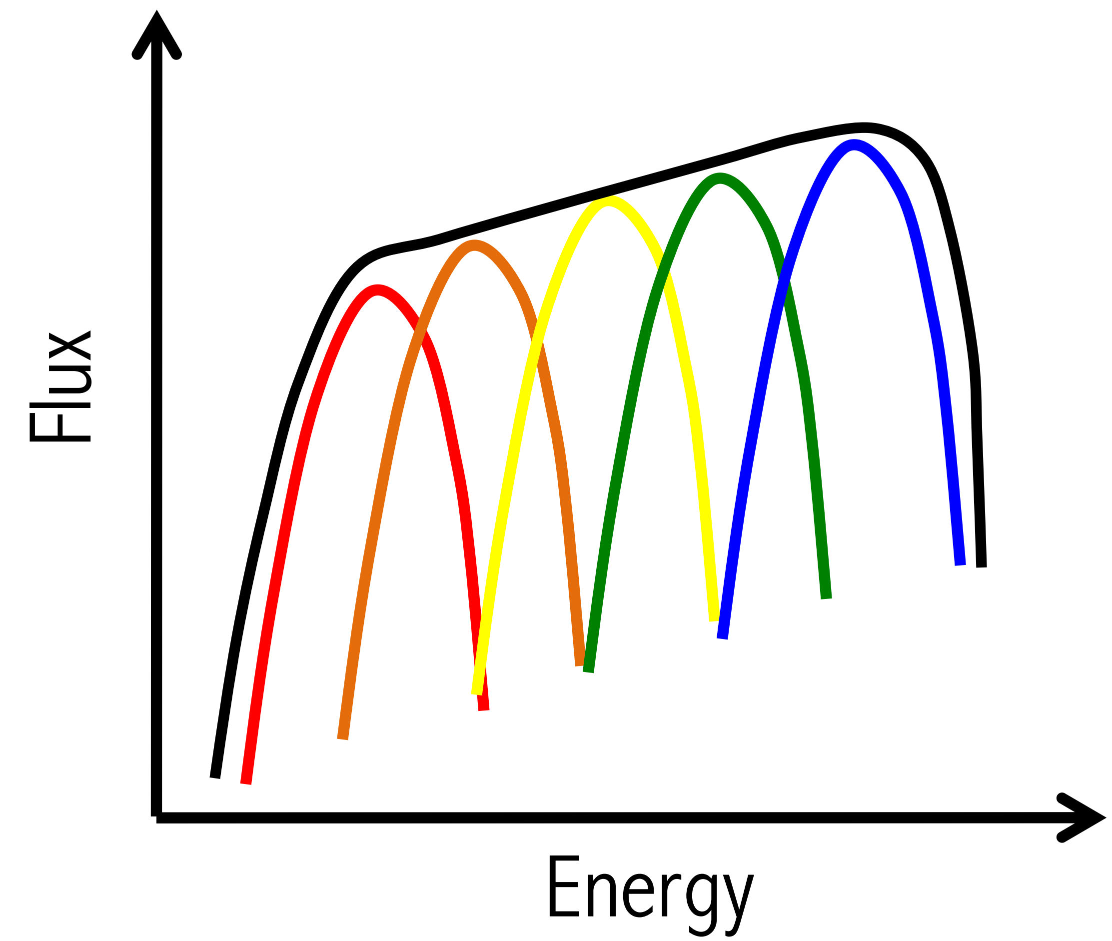

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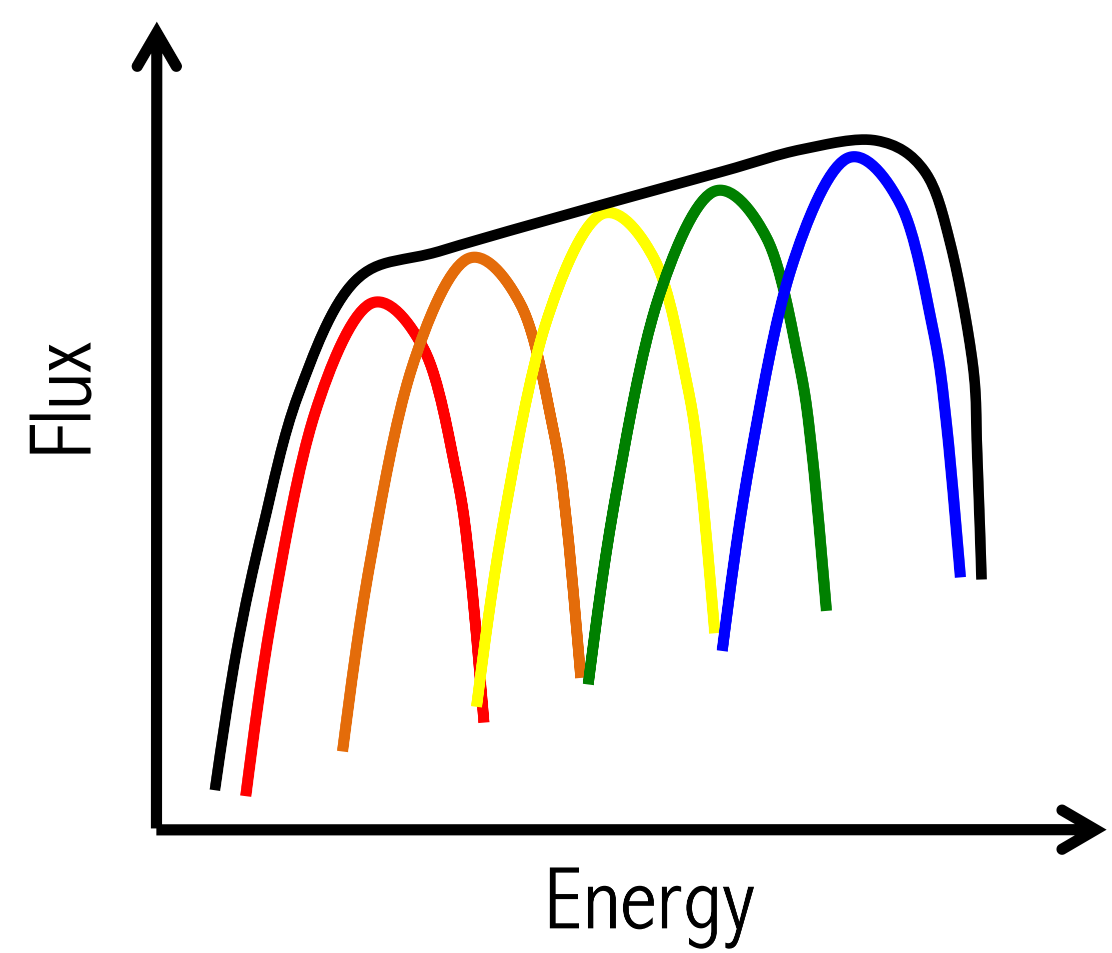

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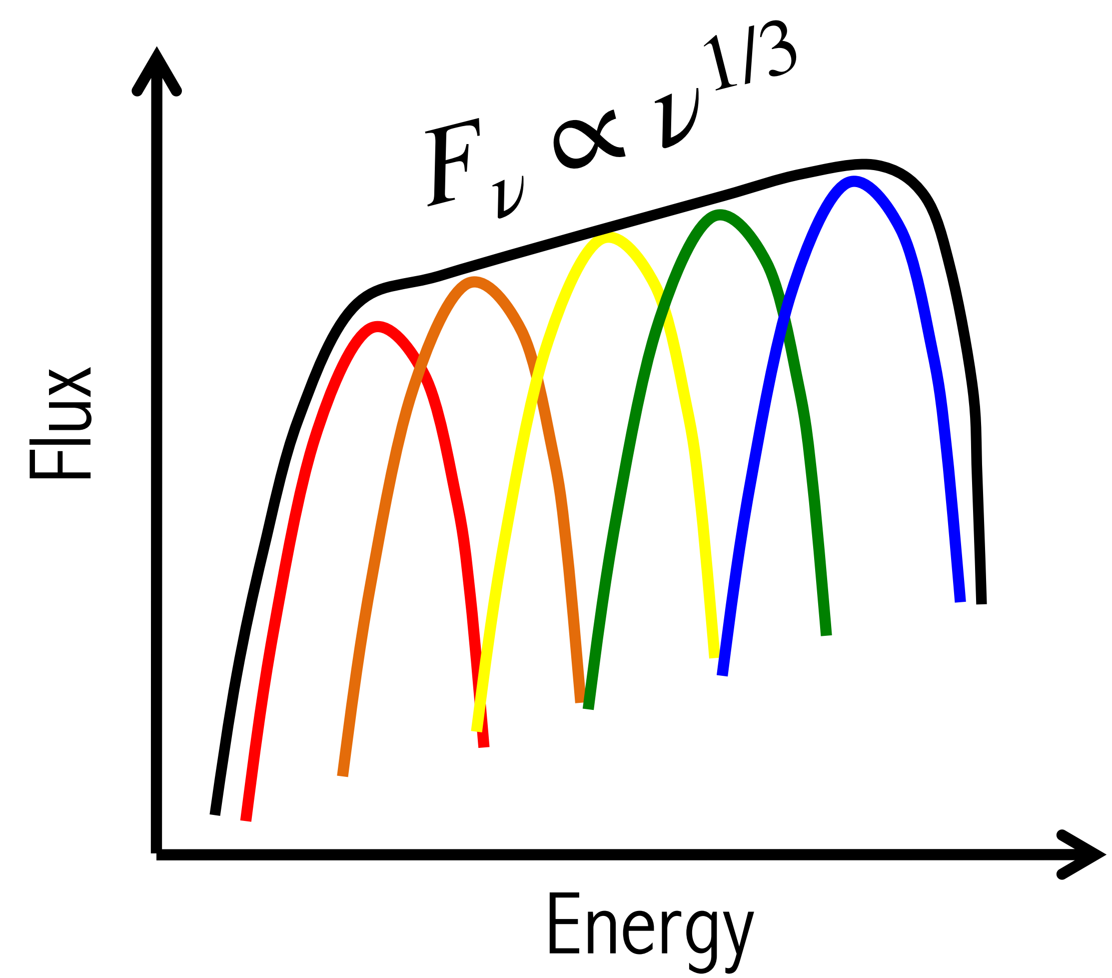

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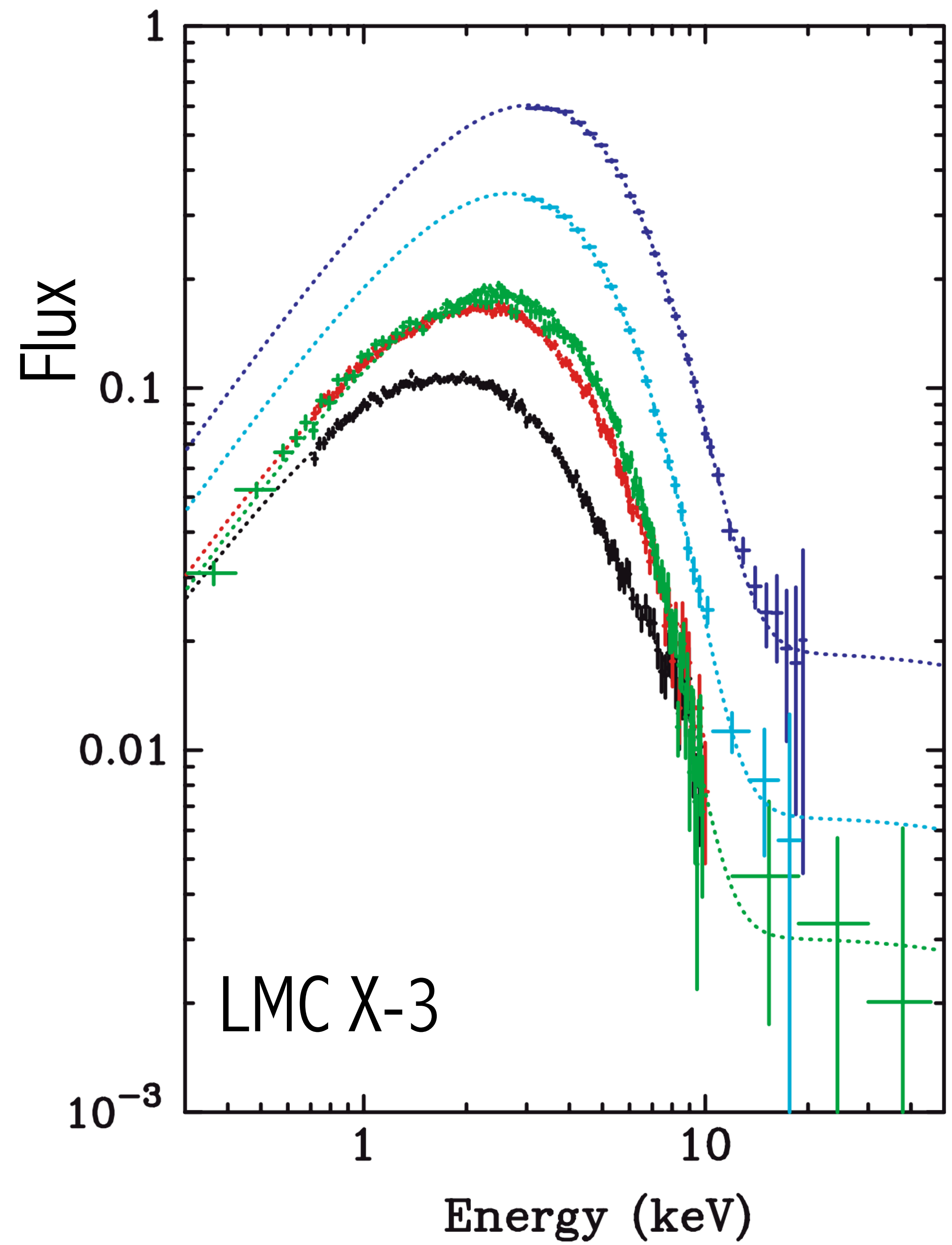

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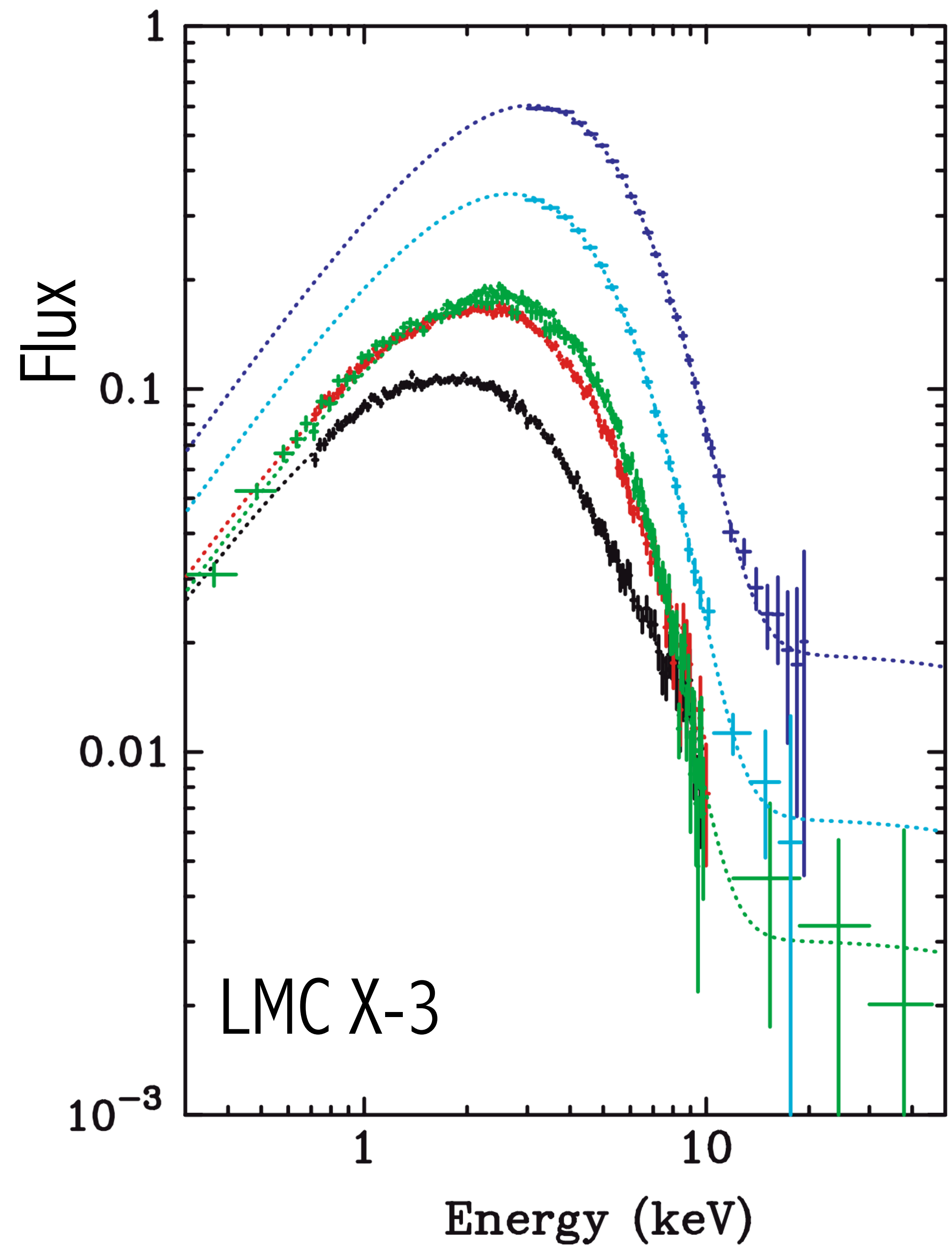

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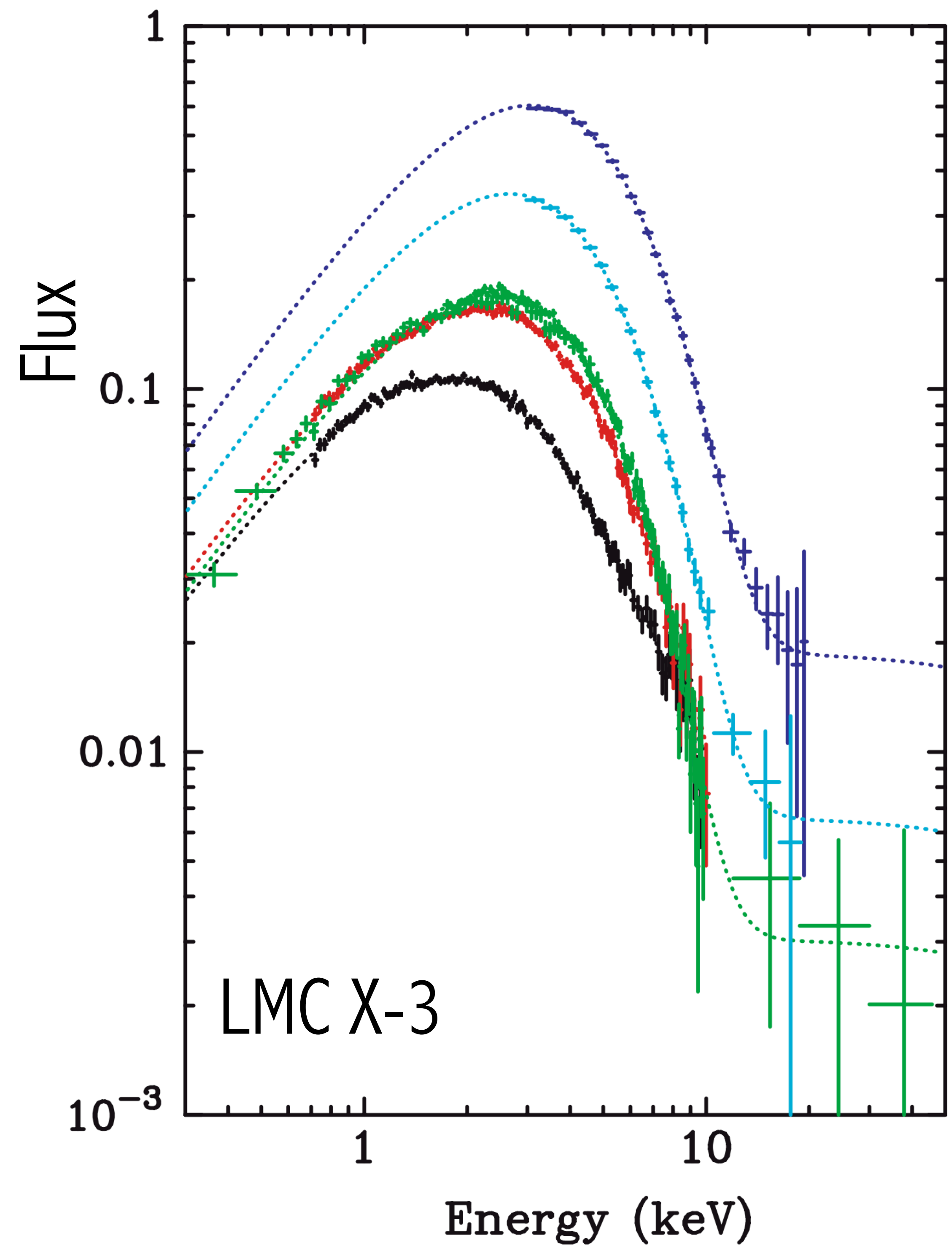

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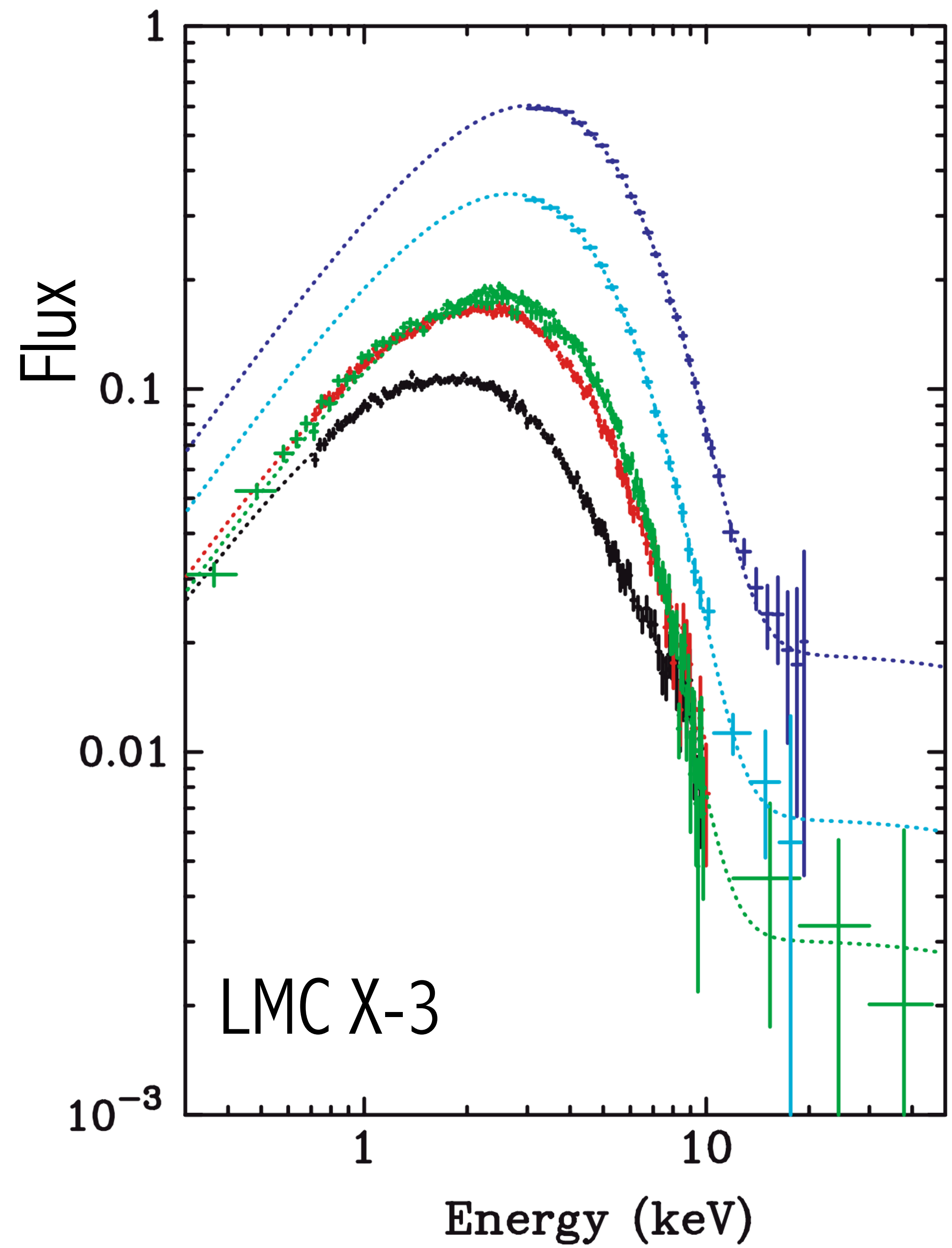


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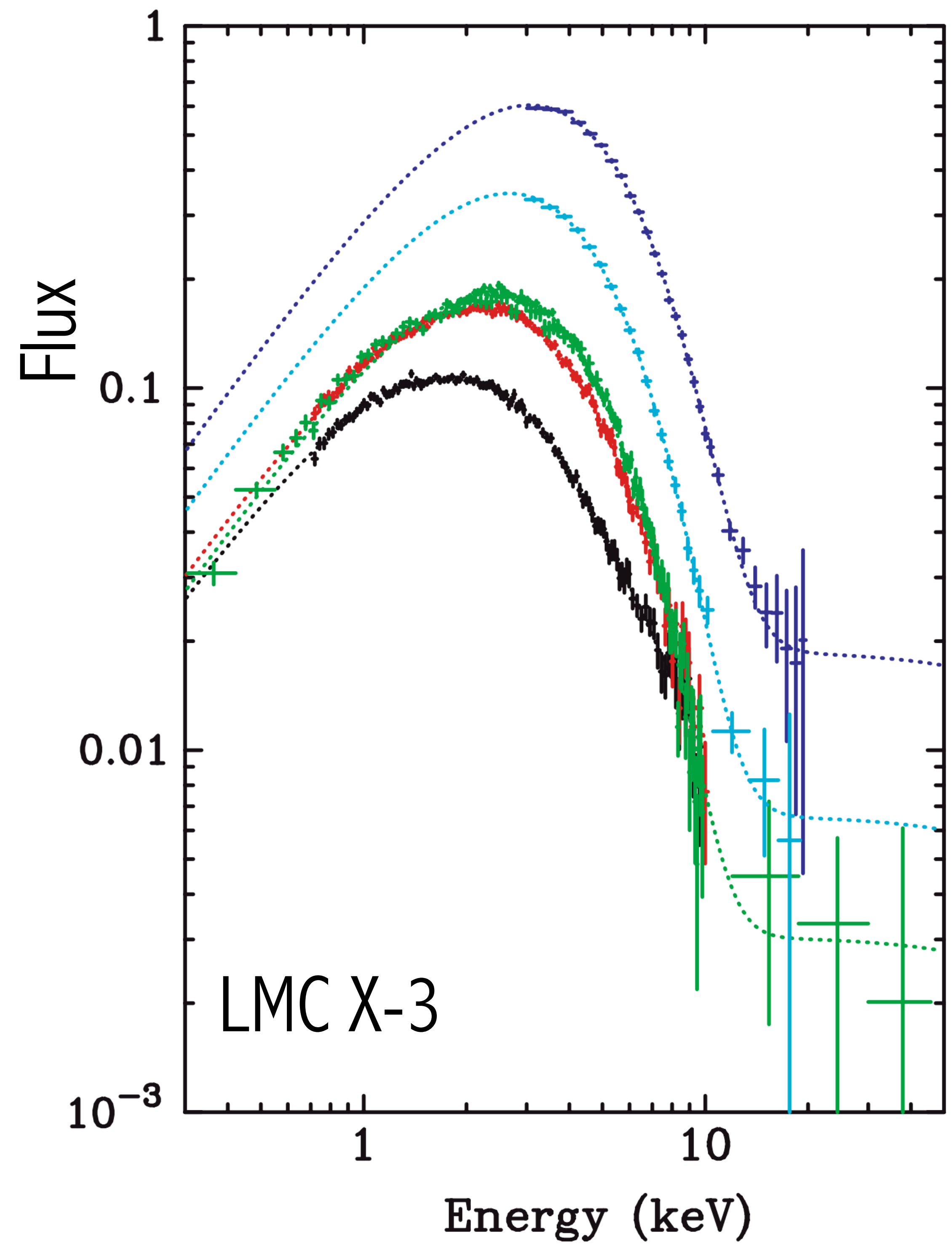


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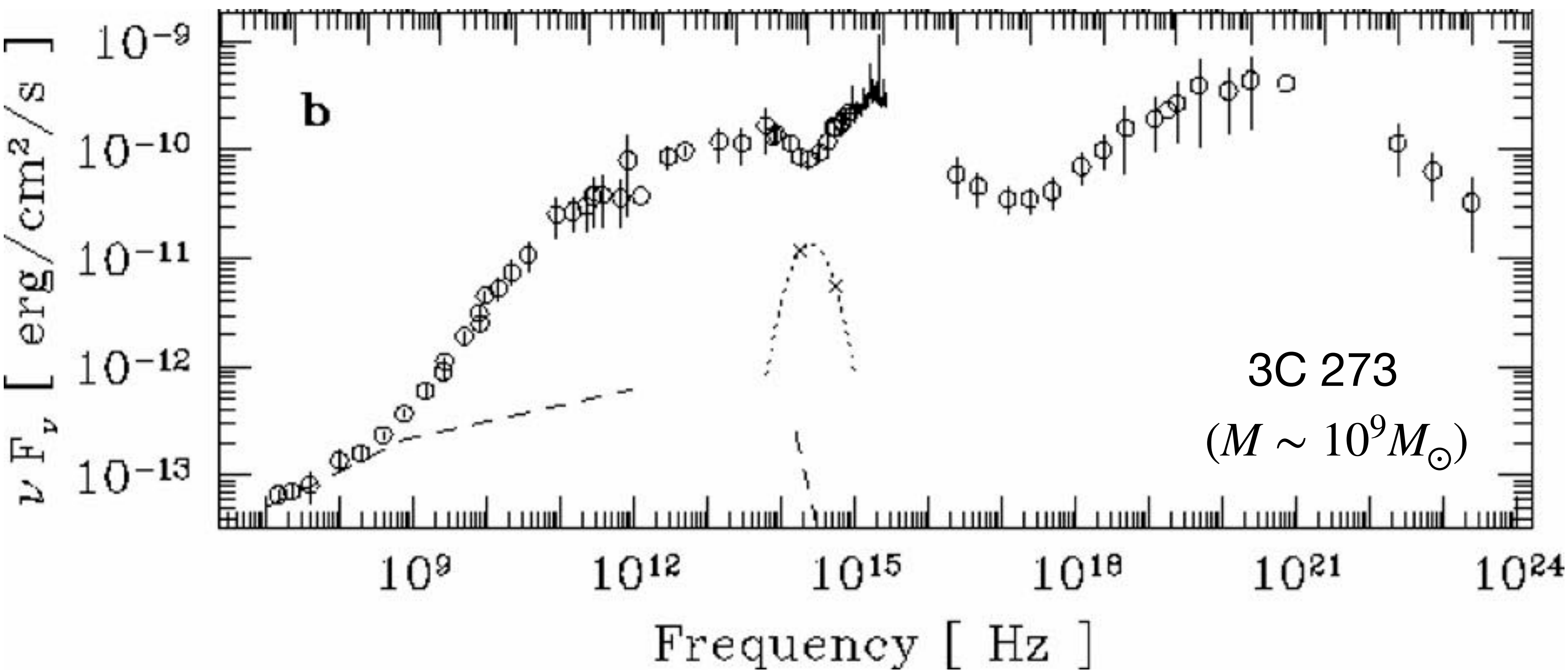

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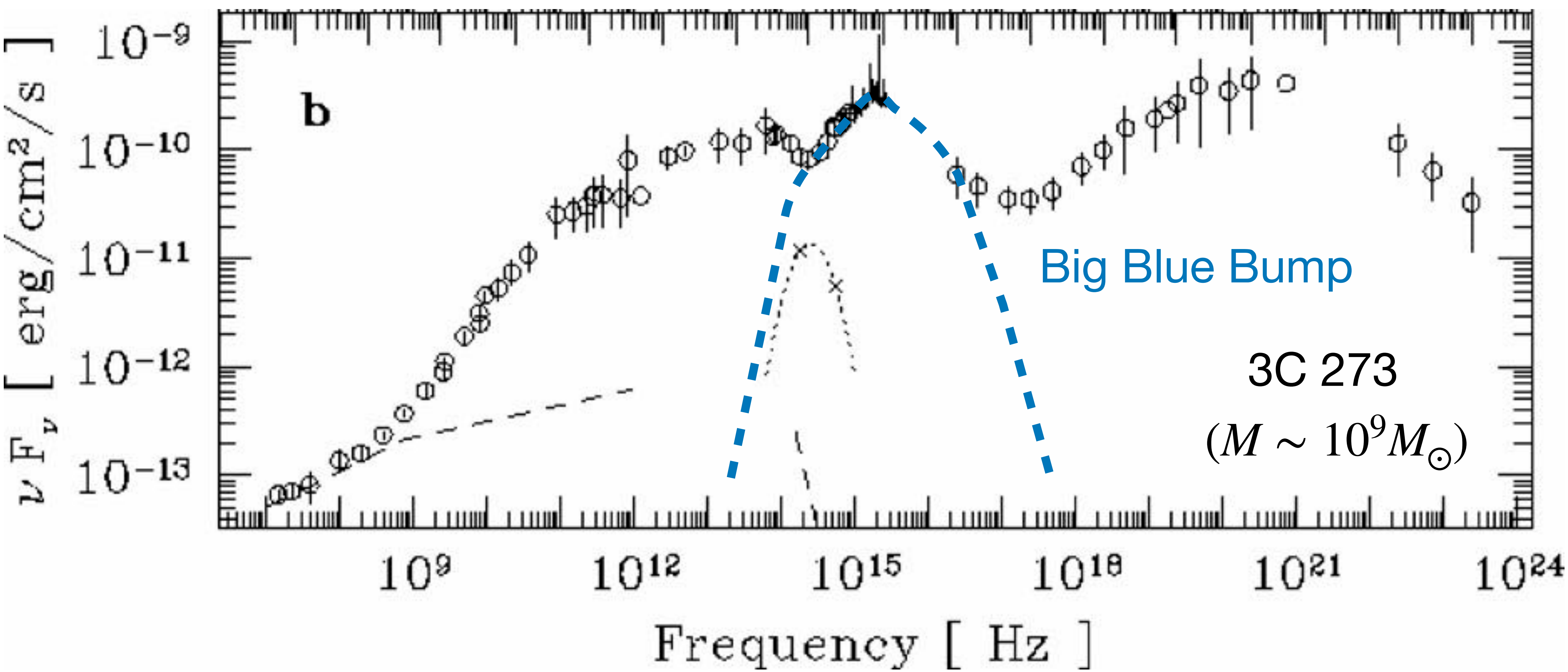

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- But see lots of other stuff, some of which we will study over the following two lectures.